6. APPLICATION OF DERIVATIVES

RATE OF CHANGE

1. Find the rate of change of the area of a circle per second with respect to its radius r when r = 5cm.
2. How fast is the area changing with respect to the radius when the radius is 3 cm ?
3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.
4. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?
5. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6cm.
6. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?
7. The radius of an air bubble is increasing at the rate of ½ cm/s. At what rate is the volume of the bubble increasing when the radius is 1cm ?
8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15cm.
9. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rate of change of

   (i) the perimeter, **[CBSE 2009]**

   (ii) the area of the rectangle.
10. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall? **[CBSE 2012]**
11. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+3)$. Determine the rate of change of volume with respect to x.
12. A man is 2 metres high, walks at a uniform speed of 6 metres per minute away from a lamp post, 5 metres high. Find the rate at which the length of his shadow increases.
13. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm ? **[CBSE 2011]**
14. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when the two equal sides are equal to the base ?
15. A man is walking at the rate of 6.5 km/hr towards the foot of a tower 120m high. At what rate is he approaching the top of the tower when he is 50m away from the tower ?
16. An inverted cone has a depth of 10cm and a base of radius 5cm. Water is poured into it at the rate of 3/2 c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4cm.
17. A water tank has the slope of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.
18. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

at which the y-coordinate is changing 8 times as fast as the x-coordinate.

19. The total cost C(x) in rupees, associated with the production of x units of an item is given by $C(x)=0.005x^3-0.02x^2+30x+5000$ Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.
20. The total revenue in rupees received from the sale of x units of a product is given by $R(x)=13x^2+26x+15$. Find the marginal revenue when x = 7.
21. The amount of pollution content added in air in a city due to x diesel vehicles is given by $P(x)=0.005x^3+0.02x^2+30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question. **[CBSE 2013]**
22. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (Marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x)=3x^2+36x+5$, find the marginal revenue, when x = 5, and write which value does the question indicate. **[CBSE 2013]**
23. A car starts from a point P at time t = 0 second and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x=t^2\left(2-\frac{t}{3}\right)$. Find the time taken by it to reach at Q and also find distance between P and Q.
24. A kite is moving horizontally at the height of 151.5 metres. If the speed of the kite is 10 m/sec, how fast is the string being let out, when the kite is 250m away from the boy who is flying the kite ? The height of the boy is 1.5m.

TANGENTS AND NORMALS

1. Find the slopes of the tangent and the normal to the curve $x^2+3y+y^2=5$ at (1, 1).
2. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}, x\neq 2$ at $x=10$.
3. Find the slope of the tangent to the curve $y=x^3-x+1$ at the point whose x-coordinate is 2.
4. Find the slope of the normal to the curve $x=a\cos^3\theta, y=a\sin^3\theta$ at $\theta=\frac{\pi}{4}$.
5. The slope of the curve $2y^2=ax^2+b$ at (1, –1) is – 1. Find a and b.
6. Find the point at which the tangent to the curve $y=\sqrt{4x-3}-1$ has its slope 2/3.
7. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to x-axis. **[CBSE 2011]**
8. Find points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are
   (i) parallel to x-axis
   (ii) parallel to y-axis
9. At what points will the tangent to the curve $y=2x^3-15x^2+36x-21$ be parallel to x-axis ? **[CBSE 2008]**
10. Find the point on the curve $y=\frac{x}{1+x^2}$, where the tangent to the curve has the greatest slope. **[2015]**
11. Find the point on the curve $y=x^3-11x+5$ at which tangent has equation $y=x-11$. **[CBSE 2012]**
12. Find a point on the curve $y=(x-2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

13. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$. **[CBSE 2012]**

14. Find the equation of tangent to the curve $y=\frac{x-7}{(x-2)(x-3)}$, at the point, where it cuts the x-axis. **[CBSE 2010]**

15. Find the equation of the tangent and normal to the curve $x=1-\cos\theta,\ y=\theta-\sin\theta\ at\ \theta=\frac{\pi}{4}$. **[CBSE 2010]**

16. Find the equation of all lines having slope –1 that are tangents to the curve $y=\frac{1}{x-1}, x\neq 1$.

17. Find the equations of all lines having slope 0 which are tangent to the curve $y=\frac{1}{x^2-2x+3}$.

18. For the curve $y=4x^3-2x^5$, find all the points at which the tangent passes through the origin. **[CBSE 2013]**

19. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$.

20. Find the equations of the tangent and normal to the parabola $y^2=4ax$ at the point $(at^2, 2at)$. **[CBSE 2013]**

21. Find the equations of the tangent and the normal at the point "t" on the curve $x=a\sin^3 t, y=b\cos^3 t$. **[CBSE 2010]**

22. Find the equations of the tangent and normal to the curve $x=a\sin^3\theta$ and $y=a\cos^3\theta$ at $\theta=\frac{\pi}{4}$. **[CBSE 2014]**

23. Prove that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at the point where it crosses the y-axis. **[CBSE 2007]**

24. Find all the tangents to the curve $y=\cos(x+y), -2\pi\leq x\leq 2\pi$ that are parallel to the line $x+2y=0$.

25. Prove that all normals to the curve $x=a\cos t+at\sin t, y=a\sin t-at\cos t$ are at a distance "a" from the origin. **[CBSE 2013]**

26. Find the equation of tangent and normal to the curve $x^{2/3}+y^{2/3}=2$ at (1, 1).

27. Prove that $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ touches the straight line $\frac{x}{a}+\frac{y}{b}=2$ for all $n\in N$, at point (a, b).

28. Prove that the curves $x=y^2$ and $xy=k$ cut at right angles if $8k^2=1$. **[CBSE 2013]**

29. Prove that the area of the triangle formed by the tangent and the normal at the point (a, a) on the curve $y^2(2a-x)=x^3$ and the line $x=2a$, is $\frac{5a^2}{4}$ sq. units.

30. Find the angle between the parabolas $y^2=4ax\ and\ x^2=4by$ at their point of intersection other than the origin.

31. Verify that the point (1, 1) is a point of intersection of the curves $x^2=y$ and $x^3+6y=7$ and show that these curves cut orthogonally at this point.

32. Find the condition for the curves $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $xy=c^2$ to intersect orthogonally.

33. Show that the curves $xy=a^2$ and $x^2+y^2=2a^2$ touch each other.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

INCREASING AND DECREASING FUNCTIONS

1. Find the intervals in which the following functions are strictly increasing or decreasing :
   (i) $10-6x-2x^2$
   (ii) $(x+1)^3(x-3)^3$ **[CBSE 2011]**
   (iii) $\dfrac{4x^2+1}{x}$
   (iv) $\log(1+x)-\dfrac{2x}{2+x}$ **[CBSE 2012]**
   (v) $x^3+\dfrac{1}{x^3}, x\neq 0$ **[CBSE 2009]**
   (vi) $\sin x+\cos x, 0\leq x\leq 2\pi$
   (vii) $\dfrac{x}{\log x}$
   (viii) $2\log(x-2)-x^2+4x+1$
   (ix) $[x(x-2)]^2$ **[CBSE 2010]**
   (x) $\dfrac{4\sin x-2x-x\cos x}{2+\cos x}$
   (xi) $f(x)=\dfrac{3}{2}x^4-4x^3-45x^2+51$ **[2014]**
   (xii) $f(x)=3x^4-4x^3-12x^2+5$ **[CBSE 2014]**
2. Prove that the function given by $f(x)=\sin x$ is
   (i) strictly increasing in $\left(0,\dfrac{\pi}{2}\right)$
   (ii) strictly decreasing in $\left(\dfrac{\pi}{2},\pi\right)$
   (iii) neither increasing nor decreasing in $(0,\pi)$.
3. Prove that $y=\dfrac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function of θ in $\left[0,\dfrac{\pi}{2}\right]$.
4. Prove that logarithmic function is strictly increasing on $(0, \infty)$.
5. On which of the following intervals is the function $f(x)=x^{100}+\sin x-1$ increasing?
   (i) $\left(0,\dfrac{\pi}{2}\right)$ (ii) $\left(\dfrac{\pi}{2},\pi\right)$
   (iii) (0, 1) (iv) (–1,1)
6. Find the values of k for which $f(x)=kx^3-9kx^2+9x+3$ is increasing on R.
7. Prove that the function f given by $f(x)=\log(\sin x)$ is strictly increasing on $\left(0,\dfrac{\pi}{2}\right)$.
8. Find the least value of 'a' such that the function f given by $f(x)=x^2+ax+1$ is strictly increasing on (1, 2).
9. Separate [0, π/2] into sub-intervals in which $f(x)=\sin^4 x+\cos^4 x$ is increasing or decreasing.
10. Separate [0, π/2] into sub-intervals in which $f(x)=\sin 3x$ is increasing or decreasing.
11. Show that $f(x)=\tan^{-1}(\sin x+\cos x)$ is a decreasing function on the interval $\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$.
12. Let I be any interval disjoint from (−1, 1). Prove that the function f given by $f(x)=x+\dfrac{1}{x}$ is strictly increasing on I.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

MAXIMA AND MINIMA

1. Find the maximum and the minimum values, if any, without using derivatives of the following functions :
   (i) $f(x) = -(x-1)^2 + 2$
   (ii) $f(x) = \sin 2x + 5$
   (iii) $f(x) = |\sin 4x + 3|$
   (iv) $f(x) = -|x+1| + 3$
   (v) $f(x) = x^3 - 1$
2. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be :
   (i) $f(x) = x^3 - 3x$
   (ii) $f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$
   (iii) $f(x) = \sin x - \cos x, 0 < x < 2\pi$
   (iv) $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$
   (v) $f(x) = \frac{1}{x^2 + 2}$
   (vi) $f(x) = x\sqrt{1-x}, x > 0$
   (vii) $f(x) = x^3(2x-1)^3$
   (viii) $f(x) = (x-2)^4(x+1)^3$
   (ix) $f(x) = x + \sqrt{1-x}, x \le 1$
   (x) $f(x) = x^3 - 6x^2 + 9x + 15$
3. Show that $f(x) = \frac{\log x}{x}$ has maximum value at x = e.
4. Find the maximum and minimum values of the function $f(x) = \frac{4}{x+2} + x$
5. The function $y = a\log x + bx^2 + x$ has extreme values at $x = 1$ and $x = 2$. Find a and b.
6. Find the absolute maximum and the absolute minimum value of the following functions in the given intervals :
   (i) $f(x) = \sin x + \cos x$ in [0, π]
   (ii) $f(x) = 4x - \frac{1}{2}x^2$ in $\left[-2, \frac{9}{2}\right]$
   (iii) $f(x) = (x-1)^2 + 3$ in [–3,1]
   (iv) $f(x) = cos^2 x + \sin x$ in $[0, \pi]$
   (v) $f(x) = 12x^{4/3} - 6x^{1/3}$ in [–1,1]
   (vi) $f(x) = \sin^2 x - \cos x,\ x \in (o, \pi)$.

   [2015]
7. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.
8. If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > o$ attains its maximum and minimum at p and q respectively such bat $p^2 = q$, then find the value of m. **[2015]**
9. Find the maximum profit that a company can make, if the profit function is given by $$p(x) = 41 - 24x - 18x^2.$$
10. Find two numbers whose sum is 24 and whose product is as large as possible.
11. Find the two positive numbers whose sum is 14 and the sum of whose squares is minimum.
12. Find the two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.
13. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.
14. Show that all the rectangles inscribed in a given fixed circle, the square has the maximum area. **[CBSE 2006, 08, 11, 13]**
15. Show that of all the rectangles of given area, the square has the smallest perimeter.

    [CBSE 2011]

16. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.
17. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
18. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$. **[CBSE 2009]**
19. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
20. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16m, BQ = 22m and AB = 20m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum. **[CBSE 2010]**
21. If the length of three sides of a trapezium other than base are equal to 10cm, then find the area of trapezium when it is maximum. **[CBSE 2010, 13]**
22. Given the sum of the perimeters of a square and a circle, show that the sum of their areas is least when one side of the square is equal to diameter of the circle. **[CBSE 2005, 11]**
23. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and other into a circle. What should be the lengths of the two pieces so that the combined area of the circle and the square is minimum ? **[CBSE 2007, 10]**
24. A wire of length 20m is to be cut into two pieces. One of the pieces will be bent into shape of a square and the other into shape of an equilateral triangle. Where the wire should be cut so that the sum of the areas of the square and triangle is minimum ?
25. A square piece of tin of side 18cm in to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume.
26. A rectangular sheet of tin 45cm by 24cm is to be made into a box without top, by cutting off squares from each corners and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum possible ?
27. A window in the form of a rectangle is surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening. **[CBSE 2011]**
28. A large window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m find the dimensions of the rectangle that will produce the largest area of the window.
29. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width. **[CBSE 2007, 10]**
30. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units. **[CBSE 2012]**
31. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm. **[CBSE 2009, 12]**
32. Show that a cylinder of a given volume which is open at the top, has minimum total surface area, provided its height is equal to the radius of its base. **[CBSE 2011, 14]**
33. Show that the height of the closed cylinder of given surface and maximum volume, is equal to the diameter of its base. **[CBSE 2012]**
34. Show that the height of a cylinder, which is open at the top, having a given surface area

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

and greatest volume, is equal to the radius of its base. **[CBSE 2010]**

35. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$. **[CBSE 2012, 13]**
36. Show that the semi-vertical angle of a cone of maximum volume and given slant height is $tan^{-1}\sqrt{2}$. **[CBSE 2011]**
37. Show that the semi – vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$. **[2014]**
38. Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $sin^{-1}\left(\frac{1}{3}\right)$.
39. Show that the volume of the largest cone that can be inscribed in a sphere or radius R is $\frac{8}{27}$ of the volume of the sphere. **[CBSE 2008, 10, 12, 13]**
40. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is 8 m³. If building of tank costs `70 per square metre for the base and `45 per square metre for sides, what is the cost of least expensive tank? **[CBSE 2009]**
41. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.
42. Prove that a conical tent of given capacity will require the least amount of canvas when the height is √2 times the radius of the base. **[CBSE 2007, 11, 13]**
43. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of that of the cone. **[CBSE 2010, 12, 13]**
44. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 tan^2\alpha$. Also, show that height of the cylinder is $\frac{h}{3}$. **[CBSE 2007, 08, 10]**
45. Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one end of the major axis. **[CBSE 2010]**
46. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ **[CBSE 2013]**
47. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$. **[CBSE 2008]**
48. Find the shortest distance of the point (0, c) from the parabola $y=x^2$, where $0\le c\le 5$
49. Find the point on the curve $x^2=8y$ which is nearest to the point (2, 4). **[CBSE 2007]**
50. Find the point on the curve $y^2=2x$ which is at a minimum distance from the point (1, 4).
51. Manufacturer can sell x items at a price of ` $\left(5-\frac{x}{100}\right)$ each. The cost price is ` $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit. **[CBSE 2009]**
52. An apache helicopter of enemy is flying along the curve given by $y=x^2+7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

APPROXIMATIONS AND ERRORS

1. Using differentials find the approximate value of each of the following upto 3 places of decimal :
   (i) $\sqrt{49.5}$ **[CBSE 2012]**
   (ii) $(82)^{\frac{1}{4}}$
   (iii) $\sqrt{0.037}$
   (iv) $(0.999)^{\frac{1}{10}}$
   (v) $(255)^{\frac{1}{4}}$
   (vi) $(0.009)^{\frac{1}{3}}$
   (vii) $(3.968)^{\frac{3}{2}}$
   (viii) $(32.15)^{\frac{1}{5}}$
   (ix) $\cos\left(\frac{11\pi}{36}\right)$
   (x) $\sin\left(\frac{22}{14}\right)$
   (xi) $\frac{1}{\sqrt{25.1}}$
   (xii) $\frac{1}{(2.002)^2}$
2. Find the approximate value of f(2.01), where $f(x)=4x^2+5x+2$.
3. Find the approximate value of $f(3.02)$ up to 2 places of decimal, where $f(x)=3x^2+5x+3$ **[2014]**
4. Find the approximate value of f(5.001), where $f(x)=x^3-7x^2+15$.
5. If the radius of a sphere is measured as 9cm with an error of 0.03cm, then find the approximate error in calculating its volume.
6. If the radius of a sphere is measured as 7m with an error of 0.02m, then find the approximate error in calculating its volume.
7. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.
8. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.
9. Find the approximate value of $\log_{10}(1005)$, given that $\log_{10} e = 0.4343$.
10. Use differentials to find the approximate value of $\log_e(4.01)$, having given that $\log_e(4)=1.3863$.
11. Using differentials, find the approximate value of $\tan 46°$, if it is being given that $1° = 0.01745\, rad$.

MEAN VALUE THEOREM

1. Verify Rolle's theorem for each of the following functions on the indicated intervals :
   (i) $f(x)=x^2+2x-8,\quad x\in[-4,2]$
   (ii) $f(x)=x^2-4x+3,\quad x\in[1,3]$ **[CBSE 2007]**
   (iii) $f(x)=(x-1)(x-2)^2,\quad x\in[1,2]$
   (iv) $f(x)=\sin 2x,\quad x\in\left[0,\frac{\pi}{2}\right]$
   (v) $f(x)=[x],\quad x\in[-2,2]$
   (vi) $f(x)=e^x\cos x,\quad x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
   (vii) $f(x)=\sin x+\cos x,\quad x\in\left[0,\frac{\pi}{2}\right]$

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

(viii) $f\left(x\right)=sin^{4}x+cos^{4}x, x\in\left[0,\frac{\pi}{2}\right]$

(ix) $f\left(x\right)=\log\left(x^{2}+2\right)-\log 3$, $x\in\left[-1,1\right]$

2. Using Rolle's theorem, find points on the curve $y=16-x^{2}$, $x\in\left[-1,1\right]$, where tangent is parallel to x-axis.
3. It is given that for the function f given by $f\left(x\right)=x^{3}+bx^{2}+ax, x\in[1,3]$. Rolle's theorem holds with $x=2+\frac{1}{\sqrt{3}}$. Find the values of a and b.
4. Verify Lagrange's mean value theorem for the following functions on the indicated intervals. In each case find a point "c" in the indicated interval as stated by the Lagrange's mean value theorem :

(i) $f\left(x\right)=x^{3}-2x^{2}-x+3$ on [0, 1]

(ii) $f\left(x\right)=x^{2}-4x-3$ on [1, 4]

(iii) $f\left(x\right)=\sqrt{x^{2}-4}$ on [2, 4]

(iv) $f\left(x\right)=x+\frac{1}{x}$ on [1, 3]

(v) $f\left(x\right)=\sin x-\sin 2x-x$ on [0, π]

(vi) $f\left(x\right)=\left[x\right]$ on $\left[-1,1\right]$

(vii) $f\left(x\right)=\frac{1}{4x-1}$ on [1, 4]

5. Find a point on the parabola $y=(x-4)^{2}$, where the tangent is parallel to the chord joining (4, 0) and (5, 1).
6. Find a point on the curve $y=x^{3}+1$ where the tangent is parallel to the chord joining (1, 2) and (3, 28).

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in