

**FULL LENGTH TEST**

**Time : 180 min.**

**MM: 100**

**General Instructions**

- (i) All questions are compulsory
- (ii) This question paper contains 29 questions.
- (iii) Question 1-4 are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 are long-answer-II type questions carrying 6 marks each.

1. If  $f(x) = x^2 + 1$ ,  $g(x) = \frac{1}{x-1}$ , find  $g \circ f(5)$ .
2. If A is a square matrix of order 3 and  $|3A| = k|A|$ , when write the value of  $k$ .
3. For what value of  $p$ , is  $(\hat{i} + j + k)p$  a unit vector?
4. For the set  $A = \{1, 2, 3\}$ , define a relation R in the set A as follows  $R = \{(1,1), (2,2), (3,3), (1,3)\}$ . Write the ordered pair to be added to R to make it the smallest equivalence relation.
5. Write the value of  $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$ .
6. If  $A = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}$  and  $B = \begin{bmatrix} -3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ , then find the matrix C such that  $A+B+C$  is a null matrix.
7. If  $y = \sin^{-1} \left[ \frac{5x + 12\sqrt{1-x^2}}{13} \right]$ , then find  $\frac{dy}{dx}$ .
8. If  $y = a \log x + bx^2 + x$  has its extreme values at  $x = -1$  and  $x = 2$ , then find  $a$  and  $b$ .
9. Evaluate:  $\int e^x \frac{x^2 + 1}{(x+1)^2} dx$
10. Form the differential equation of the family of parabolas having vertex at the origin and axis along position  $y -$  axis.
11. If  $\hat{i} + j + k, 2\hat{i} + 5j, 3\hat{i} + 2j - 3k$  and  $\hat{i} - 6j - k$  are the position vectors of the points A, B, C and D, then find the angle between  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$ . Deduce that  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are collinear.
12. If A and B are two independent events such that  $P(\overline{A} \cap B) = \frac{2}{15}$  and  $P(A \cap \overline{B}) = \frac{1}{16}$ , then find  $P(A)$  and  $P(B)$ .
13. A farmer posses 30 acre cultivated land that must be cultivated in two different mode of cultivations organic and inorganic. The yield for organic and inorganic system of cultivations is 11 quintals/acre and 14 quintals/acre respectively. Using matrix method, determine how to divide 30 acre land among two mode of cultivation to obtained yield 390 quintals. Which mode of cultivation you prefer most and why?

14. Show that the function 'f' defined by  $f(x) = \begin{cases} 3x-2, & 0 < x \leq 1 \\ 2x^2-x, & 1 < x \leq 2 \\ 5x-4, & x > 2 \end{cases}$  is continuous at  $x=2$ , but not

differentiable.

15. Find  $\frac{dy}{dx}$ , if  $y = (\cos x)^x + (\sin x)^{1/x}$

16. Show that  $y = \log(1+x) - \frac{2x}{2+x}$ ,  $x > -1$  is an increasing function of  $x$  throughout its domain.

OR

Find the equation of tangent to the curve  $y = \frac{x-7}{(x-2)(x-3)}$  at the point, where it cuts the  $x$ -axis.

17. The fuel cost for running a train is proportional to the square of the speed generated in km per hour. If the fuel costs ₹48 per hour at speed 16 km per hour and the fixed charges amount to ₹ 1200 per hour then find the most economical speed of train, when total distance covered by train is  $S$  km.

Keeping the economical status of country in mind, justify the values to be promoted in this question.

18. Evaluate:  $\int \frac{x^4 dx}{(x-1)(x^2+1)}$ .

19. Solve the differential equation  $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$ , given that  $y=1$ , when  $x=1$ .

20. Prove that, for any three vectors  $\vec{a}, \vec{b}, \vec{c}$

$$[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

21. Find the vector and Cartesian equations of the line passing through the point  $P(3, 0, 1)$  and parallel to the planes  $\vec{r} \cdot (\hat{i} + 2\hat{j}) = 0$  and  $\vec{r} \cdot (3\hat{j} - \hat{k}) = 0$ .

22. In answering a question on a MCQ test with 4 choices per question, a student knows the answer. Guesses or copies the answer. Let  $\frac{1}{2}$  be the probability that he knows the answer,  $\frac{1}{4}$  be the probability

that he guesses and  $\frac{1}{4}$  that he copies it. Assuming that a student, who copies the answer, will be correct

has the probability  $\frac{3}{4}$ , what is the probability that the student knows the answer, given that he answered it correctly?

Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts to unfair means. How would an act like the above hamper his character development in the coming years?

23. Let  $X$  denote the number of hours you study during a randomly selected school day. The probability that  $X$  can take the values  $x$ , has the following form, where  $k$  is some unknown constant.

$$P(X = x) = \begin{cases} 0.1 & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

24. Let  $f : N \rightarrow R$ , be a function defined as  $f(x) = 4x^2 + 12x + 15$ . Show that  $f : N \rightarrow S$ , where  $S$  is the range of  $f$ , is invertible. Also find the inverse of  $f$ .

25. If  $a + b + c \neq 0$  and  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ , then using properties of determinants, prove that  $a = b = c$ .

26. Using integration find the area of the triangle formed by positive  $x$ -axis and tangent and normal to the circle  $x^2 + y^2 = 4$  at  $(1, \sqrt{3})$ .

27. Evaluate:  $\int_1^4 (x^2 - x) dx$  as limit of sums.

OR

Evaluate:  $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$

28. Find the direction ratios of the normal to the plane, which passes through the points  $(1, 0, 0)$  and  $(0, 1, 0)$  and makes angle  $\frac{\pi}{4}$  with the plane  $x + y = 3$ . Also find the equation of the plane.

29. A manufacturing company makes two types of teaching aids A and B of mathematics for class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30, respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. How many pieces of type A and B should be manufactured per week to get a maximum profit? What is the maximum profit per week? Is teaching aid necessary for teaching learning process? If yes, justify your answer.