

FULL LENGTH TEST

Time : 180 min.

MM: 100

General Instructions

- (i) All questions are compulsory
- (ii) This question paper contains 29 questions.
- (iii) Question 1-4 are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 are long-answer-II type questions carrying 6 marks each.

1. State the reason for following Binary Operation $*$, defined on the set Z of integers, to be not commutative. $a * b = ab^3$.
2. Give an example of a skew symmetric matrix of order 3.
3. Find the derivative of $f(e^{\tan x})$ w.r.t. $x = 0$. It is given that $f'(1) = 5$.
4. Find the position vector of a point which divides the join the points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2 : 1.
5. Evaluate: $\tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) + \frac{\pi}{4} \right\}$.
6. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then write A^n .
7. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$.
8. The total cost $C(x)$ in rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$ Find the marginal cost when 17 units are produced.
9. Evaluate: $\int \frac{dx}{x + x \log x}$.
10. Solve: $\frac{dy}{dx} - \frac{y(x+1)}{x} = 0$.

11. Find the angle between two vectors \vec{a} and \vec{b} with magnitude 1 and 2 respectively and when $|\vec{a} \times \vec{b}| = \sqrt{3}$.
12. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.
13. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB . Hence, solve the following system of equations: $2x + y = 4, 3x + 2y = 1$.
14. If the following function is differentiable at $x = 2$, then find the values of a and b .

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 2 \\ ax + b, & \text{if } x > 2 \end{cases}$$
15. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.
16. Find the intervals in which the following function is strictly increasing or strictly decreasing. Also, find the points of local maximum and local minimum, if any.

$$f(x) = (x-1)^3(x+2)^2$$

OR

Show that the equation of normal at any point t on the curve $x = 3 \cos t - \cos^3 t$ and $y = 3 \sin t - \sin^3 t$ is $4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$.

17. Find $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$.
18. Find the general solution of the following differential equation:
 $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$
19. Solve the following differential equation:
 $(1 + y^2) dx = (\tan^{-1} y - x) dy$.
20. Show that the vectors \vec{a}, \vec{b} and \vec{c} are coplanar if $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar.
21. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{2}{3}$.
 What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem?
22. Three person A, B and C apply for a job f manager in a private company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profit of the company are 0.8, 0.5 and 0.3 respectively. If the changes does not take place, find the probability that it is due to the appointment of C.
23. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditure are in the ratio 5 : 7. If each saves ₹15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?
24. Let $f : N \rightarrow N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \rightarrow S$, where S is the range of f , is invertible. Find the inverse of f and hence find $f^{-1}(163)$.
25. Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.
26. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1)

and the lines whose equations are $x = 2y$ and $x = 3y - 3$.

27. Evaluate the following definite integral:

$$\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx.$$

OR

Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1) dx$ as a limit of a sum.

28. Find the distance of the point $3\hat{i} - 2\hat{j} + k$ from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.
29. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 hours of labour for fabricating and 1 hour for finishing. Each piece of model B requires 12 hours of labour for fabricating and 3 hours for finishing. The maximum number of labour hours, available for fabricating and for finishing, are 180 and 30 respectively. The company makes a profit of ₹8000 and ₹12000 on each piece of model A and model B respectively. How many pieces of each model should be manufactured to get maximum profit? Also, find the maximum profit.