

FULL LENGTH TEST

Time : 180 min.

MM: 100

General Instructions

- (i) All questions are compulsory
- (ii) This question paper contains 29 questions.
- (iii) Question 1-4 are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 are long-answer-II type questions carrying 6 marks each.

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$, where P is symmetric matrix and Q is skew-symmetric matrix, then find the matrix P.
2. Find the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.
3. Show that the function $f: R \rightarrow R$ given by $f(x) = \cos x, \forall x \in R$ is neither one-one nor onto.
4. Evaluate $\tan^{-1}\{\tan(-6)\}$.
5. Write the solution of the differential equation $x \frac{dy}{dx} + 2y = x^2$.
6. Find the distance of the point whose position vector is $(2\hat{i} + \hat{j} - \hat{k})$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.
7. Evaluate $\tan\left\{\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right\}$
8. Prove that the determinant $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$ is independent of θ .
9. Show that the function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ is not differentiable at $x = 2$
10. Differentiate $e^{\cos^{-1}(\sqrt{1-x^2})}$ with respect to x .
11. If $y = x^4 - 10$ and x changes from 2 to 1.99, what is the approximate change in y ? Also, find the changed value of y .

12. A company has estimated that the probabilities of success for three products introduced in the market are $\frac{1}{3}, \frac{2}{5}$ and $\frac{2}{3}$, respectively. Assuming independence, find the probability that
 - (i) The three products are successful
 - (ii) None of the products is successful.
13. Find the equation of normal to the curve $3x^2 - y^2 = 8$, which is parallel to the line $x + 3y = 4$.
14. Show that $x = 2$ is a root of the equation formed by the following determinant $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$. Hence, solve the equation.
15. If $f(x) = \begin{cases} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1}, & x \neq 0 \\ a, & x = 0 \end{cases}$ is continuous at $x = 0$, find the value of a .
16. Find the value of $\frac{dy}{dx}$, when $y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$, where a is constant.
17. Evaluate $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$.
OR
Evaluate $\int \frac{\cos 2x - \cos 2a}{\cos x - \cos a} dx$.
18. Evaluate the integral $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$.

19. Evaluate $\int_1^3 (x^2 - x + 1) dx$ using integral as the limit of sum.

20. Express the vector $\vec{a} = (5\hat{i} - 2\hat{j} + 5\hat{k})$ as sum of two vectors such that one is parallel to the vector $\vec{b} = (3\hat{i} + \hat{k})$ and the other is perpendicular to \vec{b} .

21. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

OR

Find the equation of the plane which contains the line of intersection of planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) - 15 = 0$ and is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

22. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?
OR

Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is atleast one head'.

23. Two bags A and B contain 3 red and 4 black balls and 4 red and 5 black balls, respectively. From bag A, one ball is transferred to bag B and then a ball is drawn from bag B.

The ball is found to be red in colour. Find the probability that

- (i) The transferred ball is black
- (ii) The transferred ball is red.

24. If $f : R - \{2\} \rightarrow R - \{3\}$ is defined by $f(x) = \frac{3x+1}{x-2}$, where R is the set of real numbers,

show that f is invertible and hence find the value of f^{-1} .

OR

Let $f : N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow \text{range}(f)$ is invertible. Find the inverse of f .

25. A point on the hypotenuse of a right angled triangle is at distance of a units and b units from the sides. Show that the minimum length of hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$

OR

Find the area lying above the X-axis and included between the circle $x^2 + y^2 = 8x$ and parabola $y^2 = 4x$.

26. Solve the differential equation $(x dy - y dx) y \sin\left(\frac{y}{x}\right) - (y dx + x dy) x \cos\left(\frac{y}{x}\right) = 0$

OR

Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1, x \neq 0$.

27. Find the image of the point $2\hat{i} + 3\hat{j} - 4\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3$

28. A farmer has a supply of chemical fertilisers of type A which contains 10% nitrogen and 6% phosphoric acid and type B contains 5% of nitrogen and 10% of phosphoric acid. After soil testing, it is found that atleast 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilisers of types A and B cost ₹5 and ₹8 per kg, respectively. By using LPP, find how many kilograms of each type of fertilisers should be bought to meet the requirement and cost be minimum? Solve the problem graphically.

29. To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) letters (iii) announcements. The cost for each mode per attempt is given below:

- (i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40

The number of attempts made in three villages X, Y and Z are given below:

Villages	House calls	Letters	Announcement
X	400	300	100
Y	300	250	75
Z	500	400	150