5. CONTINUITY AND DIFFERENTIABILITY

1. Prove that the function $f(x)=5x-3$ is continuous at x = 0, at $x=-3$ and at $x=5$.
2. Test the continuity of the function $f(x)$ at the origin: $f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases}$
3. Find all points of discontinuity of f, where f is defined by

   (a) $f(x)=\begin{cases}\frac{x}{|x|}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

   (b) $f(x)=\begin{cases}x+1, & \text{if } x\geq 0\\ x^2+1, & \text{if } x<0\end{cases}$

   (c) $f(x)=\begin{cases}x^2-x-6, & \text{if } x\neq 3\\ 5, & \text{if } x=3\end{cases}$
4. Discuss the continuity of the function f, where f is defined by:

   (a) $f(x)=\begin{cases}3, & \text{if } 0\leq x\leq 1\\ 4, & \text{if } 1<x<3\\ 5, & \text{if } 3\leq x\leq 10\end{cases}$

   (b) $f(x)=\begin{cases}|x|+3, & \text{if } x\leq -3\\ -2x, & \text{if } -3<x<3\\ 6x+2, & \text{if } x\geq 3\end{cases}$

   (c) $f(x)=\begin{cases}\frac{|x-a|}{x-a}, & \text{when } x\neq a\\ 1, & \text{when } x=a\end{cases}$
5. If the function $f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11, & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$ is continuous at x = 1, find the values of a and b.
6. Find the values of k, for which $f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\leq x<0\\ \frac{2x+1}{2x-1}, & \text{if } 0\leq x<1\end{cases}$ is continuous at x = 0.
7. For what value of λ, is the function $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\leq 0\\ 4x+1, & \text{if } x>0\end{cases}$ Continuous at x = 0?
8. Find the value of k, such that the function f defined by $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\frac{\pi}{2}\\ 3, & x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$.
9. Find the values of a and b such that the following function f(x) is a continuous function:

   $f(x)=\begin{cases}5, & x\leq 2\\ ax+b, & 2<x<10\\ 21, & x\geq 10\end{cases}$
10. Find the value of "a" for which the function f defined as $f(x)=\begin{cases}a\sin\left(\frac{\pi}{2}(x+1)\right), & x\leq 0\\ \frac{\tan x-\sin x}{x^3}, & x>0\end{cases}$ is continuous at x = 0.
11. Determine the values of a, b and c for which the function

    $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, & x>0\end{cases}$ may be continuous at x = 0.
12. Let $f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\frac{\pi}{2}\\ a, & \text{if } x=\frac{\pi}{2}\\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\frac{\pi}{2}\end{cases}$.

    If f(x) be a continuous function at $x=\frac{\pi}{2}$, find a and b.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

13. Examine the continuity of f, where f is defined by: $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1, & \text{if } x=0\end{cases}$.

14. Determine if f defined by $f(x)=\begin{cases}x^2\sin\frac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ is a continuous function.

15. Show that the function defined by $f(x)=\cos(x^2)$ is a continuous function.

16. Show that the function defined by $f(x)=|\cos x|$ is continuous function.

17. Examine that sin |x| is a continuous function.

18. Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

19. If function $f(x)=|x-3|+|x-4|$, then show that $f(x)$ is not differentiate at $x=3$ and $x=4$. **[2015]**

20. Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$.

21. Show that $f(x)=\begin{cases}\frac{\sin 3x}{\tan 2x}, & \text{if } x<0\\ \frac{3}{2}, & \text{if } x=0\\ \frac{\log(1+3x)}{e^{2x}-1}, & \text{if } x>0\end{cases}$

is continuous at x = 0.

22. If $f(x)=\begin{cases}\frac{2^{x+2}-16}{4^x-16}, & \text{if } x\neq 2\\ k, & \text{if } x=2\end{cases}$ is continuous at x = 2. Find k.

23. If $f(x)=\begin{cases}x^2+3x+a, & \text{for } x\leq 1\\ bx+2, & \text{for } x>1\end{cases}$ is everywhere differentiable, find the values of a and b.

24. Discuss the continuity and differentiability of $f(x)=\begin{cases}1-x, & x<1\\ (1-x)(2-x), & 1\leq x\leq 2\\ 3-x, & x>2\end{cases}$

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

DIFFERENTIATION

Chain Rule of Differentiation

Differentiate the following functions with respect to x :

1. $\tan^2 x$
2. $\sin\left(x^2+5\right)$
3. $\sec\left(\tan\sqrt{x}\right)$
4. $sin^2\left(2x+1\right)$
5. $e^{\sin\sqrt{x}}$
6. $\tan\left(5x^\circ\right)$
7. $\log_7(2x-3)$
8. $\log_x 3$
9. $\dfrac{\sin(ax+b)}{\cos(cx+d)}$
10. $\cos x^3.sin^2\left(x^5\right)$
11. $\sqrt{\dfrac{1+\sin x}{1-\sin x}}$
12. $\sqrt{\dfrac{1-x^2}{1+x^2}}$
13. $\log\left(\dfrac{\sin x}{1+\cos x}\right)$
14. $\dfrac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$
15. $\log\sqrt{\dfrac{1-\cos x}{1+\cos x}}$
16. $sin^{-1}\left(\dfrac{x}{\sqrt{x^2+a^2}}\right)$
17. $(sin^{-1}x^4)^4$
18. $\log\left(tan^{-1}x\right)$
19. $tan^{-1}\left(e^x\right)$
20. $\log\left\{x+2+\sqrt{x^2+4x+1}\right\}$
21. If $y=\log\left\{\sqrt{x-1}-\sqrt{x+1}\right\}$, show that $\dfrac{dy}{dx}=-\dfrac{1}{2\sqrt{x^2-1}}$.
22. If $y=\sqrt{x+1}+\sqrt{x-1}$, prove that $\sqrt{x^2-1}\dfrac{dy}{dx}=\dfrac{1}{2}y$.
23. If $y=\log\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)$, prove that $\dfrac{dy}{dx}=\dfrac{x-1}{2x(x+1)}$.
24. If $y=\log\sqrt{\dfrac{1+\tan x}{1-\tan x}}$, prove that $\dfrac{dy}{dx}=\sec 2x$. **[CBSE 2011]**
25. If $y=\left\{x+\sqrt{x^2+a^2}\right\}^n$, then prove that $\dfrac{dy}{dx}=\dfrac{ny}{\sqrt{x^2+a^2}}$.
26. If $y=\sqrt{\dfrac{1-x}{1+x}}$, prove that $\left(1-x^2\right)\dfrac{dy}{dx}+y=0$.
27. If $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$, prove that $\dfrac{dy}{dx}=1-y^2$.
28. If $y=\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $\left(1-x^2\right)\dfrac{dy}{dx}=x+\dfrac{y}{x}$.
29. If $y=\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log\sqrt{1-x^2}$, then prove that $\dfrac{dy}{dx}=\dfrac{\sin^{-1}x}{\left(1-x^2\right)^{\frac{3}{2}}}$.
30. If $y=x\sin^{-1}x+\sqrt{1-x^2}$, prove that $\dfrac{dy}{dx}=\sin^{-1}x$.
31. If $y=\sqrt{x^2+a^2}$, prove that $y\dfrac{dy}{dx}-x=0$.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

32. If $y=e^{x}+e^{-x}$, prove that $\frac{dy}{dx}=\sqrt{y^{2}-4}$.

33. If $y=e^{x}\cos x$, prove that

$$\frac{dy}{dx}=\sqrt{2}e^{x}\cos\left(x+\frac{\pi}{4}\right).$$

34. If $xy=4$, prove that $x\left(\frac{dy}{dx}+y^{2}\right)=3y$.

35. If $y=\frac{x}{2}\sqrt{a^{2}-x^{2}}+\frac{a^{2}}{2}\sin^{-1}\left(\frac{x}{a}\right)$. Prove that $\frac{dy}{dx}=\sqrt{a^{2}-x^{2}}$.

36. If $y=\sqrt{\frac{1+e^{x}}{1-e^{x}}}$, show that

$$\frac{dy}{dx}=\frac{e^{x}}{\left(1-e^{x}\right)\sqrt{1-e^{2x}}}.$$

Differentiation of Implicit Functions

Find $\frac{dy}{dx}$ in each of the following :

1. $xy=c^{2}$
2. $x-y=\pi$
3. $y+\sin y=\cos x$
4. $ax+by^{2}=\cos y$
5. $x^{3}+x^{2}y+xy^{2}+y^{3}=81$
6. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$
7. $\sin^{2}y+\cos xy=\pi$
8. $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$
9. $\left(x^{2}+y^{2}\right)^{2}=xy$ [CBSE 2009]
10. $\sin(xy)+\cos(x+y)=1$
11. $e^{x-y}=\log\left(\frac{x}{y}\right)$
12. $\tan^{-1}\left(x^{2}+y^{2}\right)=a$
13. If $xy=1$, prove that $\frac{dy}{dx}+y^{2}=0$.
14. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that $(1+x)^{2}\frac{dy}{dx}+1=0$. [CBSE 2011]
15. If $\sec\left(\frac{x+y}{x-y}\right)=a$, prove that $\frac{dy}{dx}=\frac{y}{x}$.
16. If $\sqrt{1-x^{2}}+\sqrt{1-y^{2}}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^{2}}{1-x^{2}}}$.
17. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and $x\neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^{2}}$. [CBSE 2012]
18. If $\cos^{-1}\left(\frac{x^{2}-y^{2}}{x^{2}+y^{2}}\right)=\tan^{-1}a$, prove that $\frac{dy}{dx}=\frac{y}{x}$.
19. If $\sin y=x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^{2}(a+y)}{\sin a}$. [CBSE 2009, 11, 12]
20. If $x\sin(a+y)+\sin a\cos(a+y)=0$, prove that $\frac{dy}{dx}=\frac{\sin^{2}(a+y)}{\sin a}$.
21. If $\cos y=x\cos(a+y)$, with $\cos a\neq\pm1$, prove that $\frac{dy}{dx}=\frac{\cos^{2}(a+y)}{\sin a}$.
22. If $y=x\sin y$, prove that $\frac{dy}{dx}=\frac{\sin y}{1-x\cos y}$.
23. If $e^{x}+e^{y}=e^{x+y}$, prove that $\frac{dy}{dx}=-\frac{e^{x}\left(e^{y}-1\right)}{e^{y}\left(e^{x}-1\right)}$.
24. If $\tan(x+y)+\tan(x-y)=1$, find $\frac{dy}{dx}$.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

25. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

26. If $\log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$, then show that $\frac{dy}{dx}=\frac{y-x}{y+x}$. [2010]

Derivatives of Inverse Trigonometric Functions

Find $\frac{dy}{dx}$ in the following:

1. $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

2. $y=\cos^{-1}\left(2x\sqrt{1-x^2}\right),\ \frac{1}{\sqrt{2}}<x<1$

3. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right),\ -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

4. $y=\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right),\ 0<x<1$

5. $y=\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right),\ -a<x<a$

6. $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right),\ 0<x<\frac{1}{\sqrt{2}}$

7. $y=\tan^{-1}\left(\frac{4x}{1-4x^2}\right),\ -\frac{1}{2}<x<\frac{1}{2}$

8. $y=\cos^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right),\ -1<x<1$

9. $y=\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right),\ -\infty<x<0$

10. $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\ 0<x<1$

11. $y=\left[\cos^{-1}\left(\frac{x-x-1}{x+x-1}\right)\right]$. [2015]

12. $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right),\ -\pi<x<\pi$

13. $y=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ [CBSE 2013]

14. If $y=\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$ and $0<x<1$, then find $\frac{dy}{dx}$. [CBSE 2010]

15. $y=\tan^{-1}\left(\frac{2a^x}{1-a^{2x}}\right), a>1, -\infty<x<0$

16. $y=\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right), x\neq 0$

17. $y=\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right), 0<x<1$

18. If $y=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, show that $\frac{dy}{dx}$ is independent of x.

19. If $y=\sin\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$, find $\frac{dy}{dx}$.

20. If $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\frac{dy}{dx}$. [CBSE 2008]

21. If $y=\cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$, find $\frac{dy}{dx}$. [CBSE 2010]

22. Differentiate $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ with respect to x. [CBSE 2013]

23. If $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, $x>0$. Prove that $\frac{dy}{dx}=\frac{4}{1+x^2}$.

Logarithmic Differentiation

Differentiate the following functions with respect to x :

1. $x^{\frac{1}{x}}$
2. $(\log x)^{\cos x}$
3. $\left(\sin^{-1} x\right)^{x}$
4. $\left(\log x\right)^{x}+\left(x\right)^{\log x}$ **[CBSE 2013]**
5. $\left(\sin x\right)^{x}+\sin^{-1}\sqrt{x}$ **[CBSE 2009, 13]**
6. $\left(x+\frac{1}{x}\right)^{x}+\left(x\right)^{\left(1+\frac{1}{x}\right)}$
7. $\left(\cos x\right)^{x}+\left(\sin x\right)^{\frac{1}{x}}$ **[CBSE 2010]**
8. $e^{\sin x}+\left(\tan x\right)^{x}$
9. $x^{\sin x}+\left(\sin x\right)^{x}$
10. $x^{x^2-3}+(x-3)^{x^2}$
11. $\left(\tan x\right)^{\log x}+\cos^{2}\left(\frac{\pi}{4}\right)$
12. $x^{x}-2^{\sin x}$
13. $x^{\cos x}+\frac{x^2+1}{x^2-1}$
14. $\left(x\cos x\right)^{x}+\left(x\sin x\right)^{\frac{1}{x}}$
15. $x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$ **[CBSE 2012]**
16. $y=x^{n}+n^{x}+x^{x}+n^{n}$
17. $y=e^{x}+10^{x}+x^{x}$
18. $y=e^{3x}.\sin 4x.2^{x}$
19. $y=\cos x.\cos 2x.\cos 3x$
20. $y=(x+3)^{2}.(x+4)^{3}.(x+5)^{4}$
21. $y=\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$
22. $y=\frac{\left(x^2-1\right)^{3}(2x-1)}{\sqrt{(x-3)(4x-1)}}$
23. Find the derivative of the function $f(x)=(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$ and hence find $f'(1)$.
24. If $x^{x}+x^{y}+y^{x}=a^{b}$, then find $\frac{dy}{dx}$. **[2015]**
25. If $x^{13}y^{7}=\left(x+y\right)^{20}$, prove that $\frac{dy}{dx}=\frac{y}{x}$
26. If $x^{m}y^{n}=1$, prove that $\frac{dy}{dx}=-\frac{my}{nx}$.
27. If $x^{m}y^{n}=\left(x+y\right)^{(m+n)}$, prove that $\frac{dy}{dx}=\frac{y}{x}$. **[2014]**
28. If $y^{x}=e^{y-x}$, prove that $\frac{dy}{dx}=\frac{(1+\log y)^{2}}{\log y}$.
29. If $\left(\sin x\right)^{y}=\left(\cos y\right)^{x}$, prove that $\frac{dy}{dx}=\frac{\log\cos y-y\cot x}{\log\sin x+x\tan y}$.
30. If $e^{x}+e^{y}=e^{x+y}$, prove that $\frac{dy}{dx}+e^{y-x}=0$.
31. If $e^{x+y}-x=0$, prove that $\frac{dy}{dx}=\frac{1-x}{x}$.
32. If $y=\left(\sin x-\cos x\right)^{\sin x-\cos x}$, find $\frac{dy}{dx}$. **[CBSE 2010]**
33. If $\left(\cos x\right)^{y}=\left(\cos y\right)^{x}$, find $\frac{dy}{dx}$.
34. If $x^{y}=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^{2}}$. **[CBSE 2010, 11, 13]**
35. Find the derivative of $f\left(x\right)$ with respect to x at $x=1$, where $f\left(x\right)=\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right]+x^{x}$ **[2015]**
36. If $y=x^{e^{-x^2}}$, find $\frac{dy}{dx}$ **[2015]**

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

Differentiation of Infinite Series

1. If $y=\sqrt{x+\sqrt{x+\sqrt{x+....to\infty}}}$, prove that $\frac{dy}{dx}=\frac{1}{2y-1}$.

2. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+......to\infty}}}$ prove that $\frac{dy}{dx}=\frac{\cos x}{2y-1}$.

3. If $y=\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)........to\infty}}$ show that $\frac{dy}{dx}=\frac{y^2}{x(2-y\log x)}$.

4. If $y=(\sin x)^{(\sin x)^{(\sin x)........to\infty}}$, prove that $\frac{dy}{dx}=\frac{y^2\cot x}{1-y\log\sin x}$.

5. If $y=(\tan x)^{(\tan x)^{(\tan x)......to\infty}}$, prove that $\frac{dy}{dx}=2$ at $x=\frac{\pi}{4}$.

6. If $y=x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+......to\infty}}}$, prove that $\frac{dy}{dx}=\frac{y}{2y-x}$.

7. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+......to\infty}}}$, prove that $(2y-1)\frac{dy}{dx}=\frac{1}{x}$.

8. If $y=\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cfrac{\cos x}{1+.....to\infty}}}}$, prove that $\frac{dy}{dx}=\frac{(1+y)\cos x+y\sin x}{1+2y+\cos x-\sin x}$.

Differentiation of Parametric Functions

Find $\frac{dy}{dx}$ in the following :

1. $x=at^2, y=2at$
2. $x=a\cos\theta, y=b\sin\theta$
3. $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$
4. $x=b\sin^2\theta, y=a\cos^2\theta$
5. $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$
6. $x=\frac{2t}{1+t^2}, y=\frac{1-t^2}{1+t^2}$
7. $x=a\left(\cos t+\log\tan\frac{t}{2}\right), y=a\sin t$
8. $x=a\left(\cos t+\frac{1}{2}\log\tan^2\frac{t}{2}\right)$ and $y=a\sin t$ **[CBSE 2011]**
9. $x=\frac{e^t+e^{-t}}{2}$ and $y=\frac{e^t-e^{-t}}{2}$
10. $x=\frac{3at}{1+t^2}$ and $y=\frac{3at^2}{1+t^2}$
11. $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$
12. If $x=2\cos\theta-\cos 2\theta$ and $y=2\sin\theta-\sin 2\theta$, prove that $\frac{dy}{dx}=\tan\left(\frac{3\theta}{2}\right)$. **[CBSE 2013]**
13. If $x=a\sin 2t(1+\cos 2t)$ and $y=b\cos 2t(1-\cos 2t)$ then find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$. **[2015]**
14. If $x=e^{\cos 2t}$ and $y=e^{\sin 2t}$, prove that $\frac{dy}{dx}=-\frac{y\log x}{x\log y}$.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

15. If $x = a\left(t + \frac{1}{t}\right)$ and $y = a\left(t - \frac{1}{t}\right)$, prove that $\frac{dy}{dx} = \frac{x}{y}$.

16. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$. **[CBSE 2012]**

17. If $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$. **[CBSE 2011]**

18. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$, prove that $\frac{dy}{dx} = 1$.

19. If $x = a\left(\frac{1+t^2}{1-t^2}\right)$ and $y = \frac{2t}{1-t^2}$, find $\frac{dy}{dx}$

20. Differentiate $(\log x)^x$ with respect to $\log x$.

21. Differentiate $\tan^{-1}\left(\frac{1-x}{1+x}\right)$ with respect to $\sqrt{1-x^2}$, if $-1 < x < 1$.

22. Differentiate $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ with respect to $\sec^{-1} x$.

23. Differentiate $\tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ with respect to $\sqrt{1+a^2x^2}$.

24. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$, $x \neq 0$.

25. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \neq 0$. **[2014]**

26. Differentiate $\cos^{-1}\left(4x^3 - 3x\right)$ with respect to $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$, if $\frac{1}{2} < x < 1$

27. Differentiate $(\cos x)^{\sin x}$ with respect to $(\sin x)^{\cos x}$.

Higher Order Derivatives

1. Find the second order derivatives of the following :
   (i) $\log(\sin x)$
   (ii) $x\cos x$
   (iii) $e^{6x}\cos 3x$
   (iv) $x^3 \log x$
   (v) $\tan^{-1} x$
   (vi) $\log(\log x)$
   (vii) $\sin(\log x)$
   (viii) $\sin^{-1} x$

2. If $y = x + \tan x$, show that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$. **[CBSE 2007]**

3. If $y = 2\sin x + 3\cos x$, show that $\frac{d^2y}{dx^2} + y = 0$.

4. If $y = \frac{\log x}{x}$, show that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$.

5. If $y = e^x \cos x$, prove that $\frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\pi}{2}\right)$. **[CBSE 2012]**

6. If $y = e^{ax}.\cos bx$, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0$. **[2015]**

7. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$. **[CBSE 2012]**

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

8. If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

9. If $y=Ae^{mx}+Be^{nx}$, show that
$$\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0.$$

10. If $y=500e^{7x}+600e^{-7x}$, show that
$$\frac{d^2y}{dx^2}=49y.$$

11. If $x=\sin\left(\frac{1}{a}\log y\right)$, show that
$$\left(1-x^2\right)y_2-xy_1-a^2y=0.$$ [CBSE 2010]

12. If $y=x^x$, prove that
$$\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0.$$ [2014]

13. If $\log y=\tan^{-1}x$, show that
$$\left(1+x^2\right)y_2+(2x-1)y_1=0.$$

14. If $y=e^x(\sin x+\cos x)$, prove that
$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0.$$ [CBSE 2009]

15. If $y=e^{a\cos^{-1}x}$, prove that
$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$$

16. If $y=\sin(\log x)$, prove that
$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$ [CBSE 2007]

17. If $y=3e^{2x}+2e^{3x}$, prove that
$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$$

18. If $y=\log(1+\cos x)$, prove that
$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\cdot\frac{dy}{dx}=0.$$

19. If $y=\left\{\log\left(x+\sqrt{x^2+1}\right)\right\}^2$, show that
$$\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=2.$$ [CBSE 2008]

20. If $y=\sin^{-1}x$, show that $\frac{d^2y}{dx^2}=\frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$.

21. If $y=A\cos(\log x)+B\sin(\log x)$, prove that
$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$

22. If $y=x\log\left(\frac{x}{a+bx}\right)$, prove that
$$x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2.$$ [CBSE 2013]

23. If $y=\log\left(x+\sqrt{x^2+a^2}\right)$, prove that
$$\left(x^2+a^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0.$$ [CBSE 2013]

24. If $y=\left(x+\sqrt{x^2+1}\right)^m$, show that
$$\left(x^2+1\right)y_2+xy_1-m^2y=0.$$ [2013]

25. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that
$$\left(1-x^2\right)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0.$$ [CBSE 2009]

26. If $y=\sqrt{x+1}-\sqrt{x-1}$, prove that
$$\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-\frac{1}{4}y=0.$$ [CBSE 2015]

27. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that
$$\left(1+x^2\right)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0.$$

28. If $y=\sin^{-1}x$, then prove that
$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0.$$ [CBSE 2012]

29. If $y=\left(\sin^{-1}x\right)^2$, prove that $\left(1-x^2\right)y_2-xy_1-2=0$.

30. If $y=e^{\tan^{-1}x}$, prove that $\left(1+x^2\right)y_2+(2x-1)y_1=0$.

31. If $y=\tan^{-1}x$, show that $\left(1+x^2\right)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$.

32. If $y=\left(\tan^{-1}x\right)^2$, then prove that $\left(1+x^2\right)^2y_2+2x\left(1+x^2\right)y_1=2$. **[CBSE 2012]**

33. If $y=\operatorname{cosec}^{-1}x, x>1$, then show that $x\left(x^2-1\right)\frac{d^2y}{dx^2}+\left(2x^2-1\right)\frac{dy}{dx}=0$. **[CBSE 2010]**

34. If $x=a\sin t$ and $y=a\left(\cos t+\log\tan\frac{t}{2}\right)$, find $\frac{d^2y}{dx^2}$. **[CBSE 2013]**

35. If $x=\cos t+\log\tan\frac{t}{2}, y=\sin t$, then find the value of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$. **[CBSE 2012]**

36. If $x=2\cos t-\cos 2t$ and $y=2\sin t-\sin 2t$, find $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{2}$.

37. If $x=a\left(1+\cos\theta\right)$ and $y=a(\theta+\sin\theta)$, prove that $\frac{d^2y}{dx^2}=-\frac{1}{a}$ at $\theta=\frac{\pi}{2}$.

38. If $x=a(\theta-\sin\theta), y=a(1+\cos\theta)$, find $\frac{d^2y}{dx^2}$. **[CBSE 2011]**

39. If $x=a\left(1-\cos^3\theta\right), y=a\sin^3\theta$, prove that $\frac{d^2y}{dx^2}=\frac{32}{27a}$ at $\theta=\frac{\pi}{6}$.

40. If $x=a\left(\cos\theta+\theta\sin\theta\right)$, $y=a(\sin\theta-\theta\cos\theta)$, prove that $\frac{d^2x}{d\theta^2}=a\left(\cos\theta-\theta\sin\theta\right)$, $\frac{d^2y}{d\theta^2}=a\left(\sin\theta+\theta\cos\theta\right)$ and $\frac{d^2y}{dx^2}=\frac{\sec^3\theta}{a\theta}$. **[CBSE 2012]**

41. If $x=a\cos\theta+b\sin\theta$ and $y=a\sin\theta-b\cos\theta$, prove that $y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$. **[CBSE 2013, 14]**

42. If $x=a\cos^3\theta, y=a\sin^3\theta$, find $\frac{d^2y}{dx^2}$. Also, find its value at $\theta=\frac{\pi}{6}$. **[CBSE 2013]**

43. If $(x-a)^2+(y-b)^2=c^2$, prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.