

4. DETERMINANTS

1. Write the minors and co - factors of each element of the first column of the following matrices:

(a) $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 5 & 0 \\ 3 & 7 & 1 \end{bmatrix}$

2. Write the minors and co - factors of each elements of the following determinants:

(a) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

3. Using co - factors of elements of third column, evaluate

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}.$$

4. Find minors and co - factors of the

elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

and verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$.

5. Evaluate the following determinants:

(a) $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$

(b) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

(c) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(d) $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

(e) $\begin{vmatrix} 0 & \sin \alpha & -\cos \alpha \\ -\sin \alpha & 0 & \sin \beta \\ \cos \alpha & -\sin \beta & 0 \end{vmatrix}$

(f) $\begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$

6. Solve the following equations:

(a) $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} x & y \\ 2 & 2x \end{vmatrix} = 0$

(c) $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$

(d) $\begin{vmatrix} a & a & x \\ a & a & a \\ b & x & b \end{vmatrix} = 0$

(e) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

(f) $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 0$

(g) $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

7. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x .

[2014]

8. What is the value of the following determinant?

$$\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}.$$

9. Write the value of the determinant.

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}.$$

10. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then

show that $|3A| = 27|A|$.

11. Prove that the determinant

$$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} \text{ is independent of } \theta.$$

12. For what value of x the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix} \text{ is singular.}$$

13. For what value of x the matrix

$$A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix} \text{ is singular?}$$

14. Evaluate: $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$.

By using properties of determinants, prove the following:

15. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

16. $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$

17. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

18. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$

19. $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

20. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

21. $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

22. $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

23. $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

24. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$

25. $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

26. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

27. $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \beta\gamma & \gamma\alpha & \alpha\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

28. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

29. $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

30. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

31. $\begin{vmatrix} x+y+2z & x & y \\ z & 2x+y+z & y \\ z & x & x+2y+z \end{vmatrix} = 2(x+y+z)^3$

32. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

33. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

$$34. \begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1) \begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$$

$$35. \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

$$36. \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

$$37. \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$$

$$38. \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

$$39. \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$$40. \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$$

$$41. \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = a^3+3a^2$$

$$42. \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

$$43. \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$44. \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$45. \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

$$46. \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

$$47. \begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a+b-c)(b+c-a)(c+a-b)$$

$$48. \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3+b^3)^2$$

$$49. \begin{vmatrix} a^2+1 & ab & ac \\ a+b & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

$$50. \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$51. \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

$$52. \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = abc + bc + ca + ab$$

$$53. \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

$$54. \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

$$55. \begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

[2014]

$$56. \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$

[2014]

$$57. \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x+1)(x+1) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$$

[2015]

58. Using the properties of determinants, solve the following for x :

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0. \quad [2015]$$

59. If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using

properties of determinants, prove that $a+b+c=0$. [2015]

60. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where $a, b,$

c are in A.P.

61. If a, b, c are positive and unequal, prove

that value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

negative.

62. If a, b, c are in A.P, find the value of

$$\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$$

63. If a, b and c are real numbers and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0. \text{ Show that}$$

$a+b+c=0$ or $a=b=c$.

64. If a, b, c are all positive and are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

Solve the following determinant equations:

$$65. \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$66. \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$67. \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

$$68. \begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

69. Prove that

$$\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix} = (2n^3 + 8n^2 + 10n + 4)(n!)^3$$

70. If ω is an imaginary cube root of unity, then

$$\text{evaluate } \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}.$$

71. If $a+b+c=0$, Solve the equation:

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

72. Prove that:

$$\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

73. Prove that $A(a, b+c)$, $B(b, c+a)$ and $C(c, a+b)$ are collinear.

74. Find the area of a triangle with vertices at the point

(a) $(1, 0), (6, 0), (4, 3)$

(b) $(-2, -3), (3, 2), (-1, -8)$

75. Find the value of K , if area of triangle is 4 sq. units and vertices are $(-2, 0), (0, 4)$ and $(0, k)$.

76. Find the equation of the line joining (3, 1) and (9, 3) using determinant.
77. If the points (a, 0), (0, b) and (1, 1) are collinear. Prove that $a + b = ab$.
78. Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find K, if D(K, 0) is a point such that area of triangle ABD is 3 sq. units.
79. Using determinants, find the value of k so that the points (k, 2-2k), (-k+1, 2k) and (-4-k, 6-2k) may be collinear.

ADJOINT AND INVERSE OF A MATRIX

1. Write minors and co - factors of the elements of following determinants:

$$(a) \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} \quad (b) \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

2. Using cofactors of elements of second row,

$$\text{evaluate } \Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$$

3. Using cofactors of elements of third

$$\text{column, evaluate } \Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}.$$

4. Write the minor of the element a_{23} in

$$\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$$

5. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

is singular.

6. Let A be square matrix of order 3×3 . Write the value of $|2A|$, where $|A| = 4$.

7. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k if

$$|2A| = K|A|.$$

8. If A is a square matrix of order 3, $|A| \neq 0$ and $|3A| = K|A|$, then write the value of K.

[2014]

9. Find the adjoint of each of the following matrices:

$$(a) \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \quad (d) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$(e) \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10. Find $A(\text{adj } A)$ for the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

11. Verify $A(\text{adj } A) = (\text{adj } A)A = |A|I$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

12. Find the inverse of each of the following matrices:

$$(a) \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

13. If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, prove that $\text{adj } A = A$.

14. Find the inverse of $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ and verify

that $A^{-1}A = I$.

15. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$. Verify that

$$(AB)^{-1} = B^{-1}A^{-1}.$$

16. Show that the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies

the equation $A^2 - 5A + 7I = 0$. Hence find A^{-1} .

17. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$. Find x and y such that

$$A^2 + xI = yA. \text{ Hence find } A^{-1}.$$

$$18. \text{ If } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

. Find $(AB)^{-1}$.

19. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Prove that

$$A^3 - 6A^2 + 5A + 11I = 0. \text{ Hence, find } A^{-1}.$$

20. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the

numbers a and b such that $A^2 + aA + bI = 0$.

21. Let A be a non-singular square matrix of order 3×3 . Then find $|\text{adj } A|$.

22. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. Verify that $A^3 = A^{-1}$.

23. Find the matrix X satisfying the matrix equation:

$$X \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}.$$

24. If $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$, then find the matrix X . [2015]

25. Find the matrix X for which

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}.$$

26. A is non-singular matrix of order 3 and $|A| = -4$. Find $|\text{adj } A|$.

27. Given a square matrix A of order 3×3 , such that $|A| = 12$, find the value of $|A \cdot \text{adj } A|$.

28. If for matrix A , $|A| = 3$, find $|5A|$, where matrix A is of order 2×2 .

29. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$. Verify that

$$(a) [\text{adj } A]^{-1} = \text{adj}(A^{-1}).$$

$$(b) (A^{-1})^{-1} = A$$

30. Using matrices, solve the following system of linear equations:

$$\begin{aligned}x - y + 2z &= 7 \\ 3x + 4y - 5z &= -5 \\ 2x - y + 3z &= 12\end{aligned}$$

31. Solve the following system of equations by matrix method:

(a) $x + y + z = 6$
 $x + 2z = 7$
 $3x + y + z = 12$

(b) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$
 $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$
 $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

(c) $x + 2y - 3z = -4$
 $2x + 3y + 2z = 2$
 $3x - 3y - 4z = 11$

32. Examine the consistency of the system of equations:

(a) $x + 3y = 5$
 $2x + 6y = 8$

(b) $3x - y - 2z = 2$
 $2y - z = -1$
 $3x - 5y = 3$

(c) $x + y + z = 1$
 $2x + 3y + 2z = 2$
 $ax + ay + 2az = 4$

(d) $5x - y + 4z = 5$
 $2x + 3y + 5z = 2$
 $5x - 2y + 6z = -1$

33. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1} [2015]

34. Find the invers of matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

and hence show that $A^{-1}A = I$ [2015]

35. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve

the system of linear equations:
 $x - 2y = 10, 2x + y + 3z = 8, -2y + z = 7$

36. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve

the following system of equations:
 $3x - 4y + 2z = -1, 2x + 3y + 5z = 7, x + z = 2$

37. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two

square matrices, find AB and hence solve the system of linear equations:

$$\begin{aligned}x - y &= 3 \\ 2x + 3y + 4z &= 17 \\ y + 2z &= 7\end{aligned}$$

38. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to

solve the system of equations:
 $x - y + 2z = 1, 2y - 3z = 1, 3x - 2y + 4z = 2$

39. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

40. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is `60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is `90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is `70. Find the cost of each item per kg by matrix method.

41. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added of two times the number of awardees

for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others. Using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which be management of the colony must include for awards.

$$(g) \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

42. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of ₹15,000 for three values 4, 3 and 2 students respectively, while school B decided to award ₹19,000 for the three values to 5, 4 and 3 students respectively. If all the three prizes together amount to ₹5,000. Then

- (a) Represent the above situation by a matrix equation and form linear equation by using matrix multiplication.
(b) Which value you prefer to be rewarded most and why?

43. Find the inverse of each of the following matrices by using elementary row transformations:

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$