9. DIFFERENTIAL EQUATIONS

1. Determine the order and degree of the following differential equations:

   (a) $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

   (b) $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

   (c) $y'' + (y')^2 + 2y = 0$

   (d) $y''' + y^2 + e^{y'} = 0$

   (e) $y''' + 2y'' + y' = 0$

   (f) $y'' + 2y' + \sin y = 0$

2. Verify that the given function is a solution of the corresponding differential equation:

   (a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6 = 0;\ y = e^{-3x}$

   (b) $\frac{d^2y}{dx^2} + y = 0;\quad y = a\cos x + b\sin x$, where a, b ∈ R

   (c) $\frac{d^2y}{dx^2} + y = 0;\quad y = a\cos x - b\sin x$, where a, b ∈ R **[CBSE 2007]**

   (d) $y = e^x + 1;\ y'' - y' = 0$

   (e) $y = x^2 + 2x + c;\ y' - 2x - 2 = 0$

   (f) $y = \cos x + c;\ y' + \sin x = 0$

   (g) $y = \sqrt{1 + x^2};\ y' = \frac{xy}{1 + x^2}$

   (h) $y = x\sin x;\ xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0 \text{ and } x > y \text{ or } x < -y)$

   (i) $xy = \log y + c;\ y' = \frac{y^2}{1 - xy}(xy \neq 1)$

3. Find the differential equation of the family of lines passing through the origin. **[2015]**

4. Form the differential equation representing the family of curves y = mx, where m is an arbitrary constant.

5. Form the differential equation representing the family of curves y = a sin (x + b), where a, b are arbitrary constant.

6. Find the differential equation representing the curve $y = cx + c^2$. **[2015]**

7. Form the differential equation representing the family of curves y = A cos (x + B), where A and B are parameters. **[CBSE 2007]**

8. Write a differential equation for $y = A\cos\alpha x + B\sin\alpha x$, where A and B are arbitrary constants. **[2015]**

9. Form the differential equation representing the family of curves y = a sin (bx + c), where a and c are being parameters.

10. Find the differential equation of all circles touching the

    (a) x – axis at the origin.

    (b) y – axis at the origin.

11. Form the differential equation corresponding to $y^2 = a(b - x)(b + x)$ by eliminating parameters a and b.

12. Form the differential equation of the family of curves, $x = A\cos nt + B\sin nt$, where A and B are arbitrary constant. **[CBSE 2007]**

13. Form the differential equation of the family of curves represented by $y^2 = (x - c)^3$.

14. Form the differential equation of the family of curves represented by the equation (a being the parameter).

    (a) $(2x + a)^2 + y^2 = a^2$

    (b) $(2x - a)^2 - y^2 = a^2$

    (c) $(x - a)^2 + 2y^2 = a^2$

15. Form the differential equation representing the given family of curves by eliminating arbitrary constants a and b.

    (a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $y^2 = a(b^2 - x^2)$

    (c) $y = ae^{3x} + be^{-2x}$ (d) $y = e^{2x}(a + bx)$

    (e) $y = e^x(a\cos x + b\sin x)$

16. Form the differential equation corresponding to $y^2 - 2ay + x^2 = a^2$ by eliminating a. **[CBSE 2005]**

17. Form a differential equation of the family of circles touching the x – axis at origin.

18. Form a differential equation of the family of circles touching the y – axis at origin.
19. Form a differential equation of the family of circles having centre on y – axis and radius 3 units.
20. Form a differential equation of the family of circles in the first quadrant and touching the coordinate axes.
21. Form a differential equation of the family of circles in the second quadrant and touching the coordinate axes.
22. Form a differential equation representing the family of circles given by $(x-a)^2+2y^2=a^2$, where a is an arbitrary constant.
23. Obtain the differential equation of al circles of radius r. **[CBSE 2010]**
24. Show that the differential equation representing one parameter family of curves $(x^2-y^2)=c(x^2+y^2)^2$ is $(x^3-3xy^2)dx=(y^3-3x^2y)dy$.
25. Form the differential equation representing the family of ellipses having foci on x – axis and centre at the origin. **[CBSE 2007]**
26. Form the differential equation representing the family of ellipses having foci on y – axis and centre at the origin.
27. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x – axis.
28. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of y – axis.
29. Form the differential equation of the family of hyperbolas having foci on x – axis and centre at origin.
30. Find the differential equation for all the straight lines, which are at a unit distance from the origin. **[2015]**
31. For each of the following differential equation, find the general solution:
    (a) $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$
    (b) $\frac{dy}{dx}=\sqrt{4-y^2}, -2<y<2$
    (c) $\frac{dy}{dx}=(1+x^2)(1+y^2)$
    (d) $y\log y\,dx-x\,dy=0$
    (e) $\frac{dy}{dx}=\sin^{-1}x$
    (f) $\frac{dy}{dx}+y=1,\ y\neq 1$
32. Find a particular solution for each of the following differential equations:
    (a) $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0, \text{when } x=1$
    (b) $(x+y)dy+(x-y)dx=0; y=1, \text{when } x=1$
    (c) $x^2dy+(xy+y^2)dx=0; y=1, \text{when } x=1$
    (d) $x(x^2-1)\frac{dy}{dx}=1; y=0, \text{when } x=2$
    (e) $\cos\left(\frac{dy}{dx}\right)=a, y=1 \text{ when } x=0$
    (f) $\frac{dy}{dx}=y\tan x, y=1 \text{ when } x=0$
    (g) $e^x\sqrt{1-y^2}\,dx+\frac{y}{x}\,dy=0,\ y=1$ when $x=0.$ **[2014]**
    (h) $x^2dy=(2xy+y^2)dx,\ y=1$ when $x=1.$ **[2015]**
    (i) $(1+x^2)\frac{dy}{dx}=(e^{m\tan^{-1}x}-y),\ y=1$ when $x=0.$ **[2015]**

Solve the following differential equations:

33. $\frac{dy}{dx}=\frac{x}{x^2+1}$
34. $(e^x+e^{-x})\frac{dy}{dx}=(e^x-e^{-x})$
35. $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ **[CBSE 2015]**

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

36. $(x+2)\frac{dy}{dx}=x^2+4x-9,\ x\neq-2$

37. $(x-y)\frac{dy}{dx}=x+2y$

38. $y\,dx+x\log\left(\frac{y}{x}\right)dy-2x\,dy=0$

39. $\frac{dy}{dx}-y=\cos x$

40. $x\frac{dy}{dx}+2y=x^2, x\neq 0$

41. $\frac{dy}{dx}+2y=\sin x$

42. $\frac{dy}{dx}+\left(\frac{y}{x}\right)=x^2$

43. $\frac{dy}{dx}+(\sec x)y=\tan x\left(0<x<\frac{\pi}{2}\right)$

44. $x\frac{dy}{dx}+2y=x^2\log x$

45. $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

46. $(1+x^2)dy+2xy\,dx=\cot dx$

47. $(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$ **[2014]**

48. $(x+y)\frac{dy}{dx}=1$

49. $y\,dx+(x-y^2)dy=0$

50. $(x+3y^2)\frac{dy}{dx}=y,\ y>0$

51. $\frac{dy}{dx}=\sin^3 x\cos^2 x+xe^x$

Solve the following differential equations :

52. $\frac{dy}{dx}=\frac{1}{\sin^4 x+\cos^4 x}$

53. $\frac{dy}{dx}=\frac{3e^{2x}+3e^{4x}}{e^x+e^{-x}}$

54. $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ **[CBSE 2002]**

55. $(1+x^2)\frac{dy}{dx}-x=2\tan^{-1}x$ **[CBSE 2007]**

56. $(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x$ **[CBSE 2010]**

57. $x(x^2-1)\frac{dy}{dx}=1, y(2)=0$ **[CBSE 2012]**

58. $\frac{dy}{dx}+\frac{1+y^2}{y}=0, y\neq 0$

59. $\frac{dy}{dx}=\frac{1+y^2}{y}, y\neq 0$

60. $\frac{dy}{dx}=\sin^2 y$

61. $\frac{dy}{dx}=\frac{1-\cos 2y}{1+\cos 2y}$

62. $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$ **[CBSE 2007]**

63. $e^x\sqrt{1-y^2}\,dx+\frac{y}{x}dy=0$ **[CBSE 2012]**

64. Solve the following differential equation $(1+e^{2x})dy+1(1+y^2)e^x dx=0$ given that when $x = 0, y = 0.$ **[CBSE 2004, 2005]**

65. Solve the following differential equation $(1+y^2)(1+\log x)dx+xdy=0$ given that when $x = 1, y = 1.$ **[CBSE 2011]**

66. Solve the following differential equations:

(a) $\frac{dy}{dx}=1+x+y+xy$

(b) $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$ **[CBSE 2002]**

67. Solve:

(a) $(x^2-yx^2)dy+(y^2+x^2y^2)dx=0$ **[2012]**

(b) $3e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

68. Solve:

(a) $\sin^3 x\frac{dx}{dy}=\sin y$ (b) $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

69. Solve:

(a) $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ (b) $\frac{dx}{dy}=\frac{1+y^2}{1+x^2}$

70. Solve $\frac{dy}{dx}=y\sin 2x$. It being given that $y(0)=1$ **[CBSE 2004]**

71. Find the equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ whose differential equation is $\sin x\cos y\,dx+\cos x\sin y\,dy=0$.

72. Solve the following initial value problems:

(a) $(x+1)\frac{dy}{dx}=2e^{-y}-1, y(0)=0$ **[CBSE 2012]**

(b) $y-x\frac{dy}{dx}=2\left(1+x^2\frac{dy}{dx}\right), y(1)=1$

73. solve the following differential equations:

(a) $x\cos y\,dy=\left(xe^x\log x+e^x\right)dx$ **[CBSE 2007]**

(b) $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ **[CBSE 2010]**

(c) $(y+xy)dx+\left(x-xy^2\right)dy=0$ **[CBSE 2002]**

(d) $\frac{dy}{dx}=1-x+y-xy$ **[CBSE 2002C]**

(e) $y\left(1-x^2\right)\frac{dy}{dx}=x\left(1+y^2\right)$ **[CBSE 2007]**

74. Solve:

(a) $\frac{dy}{dx}=y\tan x, y(0)=1$ **[CBSE 2010]**

(b) $\frac{dy}{dx}=1+x^2+y^2+x^2y^2, y(0)=1$ **[CBSE 2012]**

(c) $xy\frac{dy}{dx}=(x+2)(y+2), y(1)=-1$ **[CBSE 2012]**

75. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ is given by $x+y+1=A(1-x-y-2xy)$, where A is a parameter.

76. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)=3x+4y$ given that y = 0, when x = 0.

77. In a bank principal increases at the rate of 5% per year. In how many years `1000 double itself.

78. Find the equation of the curve passing through the point (0, –2) given that at any points (x, y) on the curve the product of the slope of its tangent and y coordinate of the point is equal to the x – coordinate of the point.

79. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $\frac{dy}{dx}=e^x\sin x$.

80. Find the equation of the curve passing through the point (1, 1) whose differential equation is $x\,dy=\left(2x^2+1\right)dx,\ x\neq 0$.

81. Find the equation of the curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

82. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2+y^2}{2xy}$ is given by $x^2-y^2=Cx$.

83. Find the equation of the curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x – coordinate and the product of the x – coordinate and y – coordinate of that point.

84. Find the equation of the curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

85. Find the equation of the curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

86. The slope of the tangent to the curve at any point is reciprocal of twice the ordinate at that point. The curve passes through the point (4, 3). Determine its equation.

87. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$. Find the solution of the curve passing through the point (1, –1). **[CBSE 2012]**

88. At any point (x, y) of a curve the slope of the tangent is twice the slope of the line segment joining the point of contact to the

point (−4, −3). Find the equation of the curve given that it passes through (−2, 1).

89. The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds.

90. In a bank principal increases at the rate of r% per year. Find the value of r, if `100 double itself in 10 years. $(\log_e 2 = 0.6931)$.

91. In a bank principal increases at the rate of 5% per year. An amount of `1000 is deposited with this bank, how much will it worth after 10 years. $\left(e^{0.5} = 1.648\right)$.

92. In a culture bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200000, if the rate growth of bacteria is proportional to the number present?

93. Show that the differential equation $(x-y)\frac{dy}{dx} = x+2y$ is homogenous and solve it.

94. Show that the differential equation $2xy\frac{dy}{dx} = x^2+3y^2$ is homogeneous and solve it. **[2015]**

95. Solve the following initial value problems:

(a) $(x+y+1)^2 dy = dx, y(-1) = 0$

(b) $(x-y)(dx+dy) = dx-dy,\ y(0) = -1$

96. Solve the differential equation $\left(x^2-y^2\right)dx + 2xydy = 0$; given that y = 1 when x = 1. **[CBSE2008]**

97. Solve the following differential equation, given that $y = 0$, when $x = \frac{\pi}{4}$.

$\sin 2x\frac{dy}{dx} - y = \tan x$ **[CBSE 2015]**

98. Solve: $x^2 y\,dx - \left(x^3+y^3\right)dy = 0$ **[CBSE 2002]**

99. Solve: $\left(x^2+xy\right)dy = \left(x^2+y^2\right)dx$ **[CBSE 2005]**

100. Solve: $\left(3xy+y^2\right)dx + \left(x^2+xy\right)dy = 0$ **[CBSE 2008]**

101. Solve: $xdy - ydx = \sqrt{x^2+y^2}\,dx$ **[CBSE 2005, 2011]**

102. Solve:

$y\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}dx - x\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}dy = 0$ **[CBSE 2010]**

103. Solve: $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ **[CBSE 2002]**

104. Solve: $2ye^{x/y}dx + \left(y-2xe^{x/y}\right)dy = 0$. **[CBSE 2012]**

105. Solve each of the following initial value problems:

(a) $2x^2\frac{dy}{dx} - 2xy + y^2 = 0, y(e) = e$ **[CBSE 2012]**

(b) $2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y(1) = 2$

106. Solve the following differential equations:

(a) $x^2 dy + y(x+y)dx = 0$

(b) $\frac{dy}{dx} = \frac{y-x}{y+x}$ **[CBSE 2004]**

(c) $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

(d) $xy\log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2\log\left(\frac{x}{y}\right)\right\}dy = 0$ **[CBSE 2010]**

(e) $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

(f) $(x-y)\frac{dy}{dx} = x+2y$ **[CBSE 2010]**

107. Solve each of the initial value problems:

(a) $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0, y(1) = 0$ **[CBSE 2009]**

(b) $\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + x\,dy = 0$ **[CBSE 2011]**

(c) $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0, y(2) = \pi$ **[CBSE 2012]**

108. Solve the differential equation:

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

$\frac{dy}{dx}-\frac{y}{x}=2x^2, x>0$

[CBSE 2004, 2007, 2011]

109. Solve the differential equation:

$x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x, x>0$ **[CBSE 2010]**

110. Solve the differential equation:

$\left(x^2-1\right)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}$ **[CBSE 2010]**

111. Solve: $\frac{dy}{dx}+y\sec x=\tan x, \left(0\le x\le\frac{\pi}{2}\right)$

[CBSE 2008, 2012]

112. Solve: $\cos^2 x\frac{dy}{dx}+y=\tan x, \left(0\le x<\frac{\pi}{2}\right)$

[CBSE 2008, 2011]

113. Solve: $\left(x^2+1\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

[CBSE 2010]

114. Solve:

(a) $x\frac{dy}{dx}+y-x+xy\cot x=0, x\ne 0$

[CBSE 2011, 2012]

(b) $\left(1+x^2\right)dy+2xy\,dx=\cot x\,dx, x\ne 0$

[CBSE 2012]

115. Solve: $y\,dx-\left(x+2y^2\right)dy=0$

116. Solve: $y\,dx+\left(x-y^3\right)dy=0$ **[CBSE 2011]**

117. Solve: $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1, x\ne 0.$

[CBSE 2012]

118. Solve each of the following initial value problems:

(a) $(x-\sin y)dy+(\tan y)dx=0, y(0)=0$

(b) $\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy, y(0)=0$

(c) $\frac{dy}{dx}+y\cot x=4x\,\mathrm{cosec}\,x, y\left(\frac{\pi}{2}\right)=0$

[CBSE 2012]

(d) $\frac{dy}{dx}+2y\tan x=\sin x; y=0 \text{ when } x=\frac{\pi}{3}$

(e) $\frac{dy}{dx}-3y\cot x=\sin 2x; y=2 \text{ when } x=\frac{\pi}{2}.$

119. Solve the following differential equations:

(a) $4\frac{dy}{dx}+8y=5e^{-3x}$ **[CBSE 2007]**

(b) $\frac{dy}{dx}+2y=6e^x$ **[CBSE 2007C]**

(c) $\frac{dy}{dx}+\frac{4x}{x^2+1}y+\frac{1}{\left(x^2+1\right)^2}=0$ **[CBSE 2005]**

(d) $\frac{dy}{dx}+y=\sin x$

(e) $\frac{dy}{dx}+2y=\sin x$

(f) $\left(1+x^2\right)\frac{dy}{dx}+y=\tan^{-1}x$ **[CBSE 2009]**

(g) $\frac{dy}{dx}+y\cot x=x^2\cot x+2x$ **[CBSE 2005]**

(h) $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$ **[CBSE 2002]**

(i) $\left(1+x^2\right)\frac{dy}{dx}-2xy=\left(x^2+2\right)\left(x^2+1\right)$

[CBSE 2005]

(j) $\frac{dy}{dx}+2y=x\cos x$

(k) $\frac{dy}{dx}-y=xe^x$ **[CBSE 2002]**

(l) $\frac{dy}{dx}+2y=xe^{4x}$ **[CBSE 2002C]**

120. Solve the differential equation: $\left(x+2y^2\right)\frac{dy}{dx}=y$, give that when x = 2, y = 1.

121. Solve the differential equation $\frac{dx}{dy}+y\cot x=2\cos x$, given that $y=0$, when $x=\frac{\pi}{2}$. **[2014]**

122. Solve the differential equation $\left(x^2-yx^2\right)dy+\left(y^2+x^2y^2\right)dx=0$, given that $y=1$, when $x=1$ **[2014]**

123. Solve the differential equation:

$x\log x\frac{dy}{dx}+y=2\log x$ **[CBSE 2009, 2015]**

124. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\left(x\ne 0\right)$.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

125. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$.

126. Solve the differential equation: $\left(y + 3x^2\right)\frac{dx}{dy} = x$. **[CBSE 2011]**

127. Write the integrating factor of the following differential equation: $\left(1 + y^2\right)dx - \left(\tan^{-1} y - x\right)dy = 0.$ **[2015]**

128. Find the product of the order and degree of the following differential equation:
$$x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$$ **[2014]**

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in