

7A. INDEFINITE INTEGRALS

Evaluate the following :

1. $\int \frac{1}{x^5} dx$

2. $\int \frac{1}{x^{\frac{3}{2}}} dx$

3. $\int \frac{1}{\sqrt[3]{x^2}} dx$

4. $\int \log_x x dx$

5. $\int (4e^{3x} + 1) dx$

6. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$

7. $\int (ax^2 + bx + c) dx$

8. $\int (2x^2 + e^x) dx$

9. $\int (1-x)\sqrt{x} dx$

10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

11. $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

12. $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

13. $\int \frac{1}{a^x b^x} dx$

14. $\int (2x^2 - 3\sin x + 5\sqrt{x}) dx$

15. $\int \sec x (\sec x + \tan x) dx$

16. $\int \frac{\sec^2 x}{\cos ec^2 x} dx$

17. $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

18. $\int \frac{\cos 2x + 2\sin^2 x}{\sin^2 x} dx$

19. $\int \frac{2\cos^2 x - \cos 2x}{\cos^2 x} dx$

20. $\int \sqrt{\frac{1 + \cos 2x}{2}} dx$

21. $\int \frac{e^{6\log_e x} - e^{5\log_e x}}{e^{4\log_e x} - e^{3\log_e x}} dx$

22. $\int \sqrt{1 - \sin 2x} dx$

23. $\int \tan^2 x dx$

24. $\int \frac{1}{\sin^2 x \cos^2 x} dx$ [2014]

25. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

26. $\int (2 \tan x - 3 \cot x)^2 dx$

27. $\int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx\right) dx$

28. $\int \frac{(1+x)^3}{\sqrt{x}} dx$

29. $\int \frac{x^6 + 1}{x^2 + 1} dx$

30. $\int \frac{x^{\frac{1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}} dx$

31. $\int \frac{\sin^2 x}{1 + \cos x} dx$

32. $\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x} dx$

33. $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$

34. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ [CBSE 2013]

35. $\int \frac{1}{1 + \sin x} dx$

36. $\int \frac{\sin x}{1 + \sin x} dx$

37. $\int \frac{\sec x}{\sec x + \tan x} dx$

38. $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$ [CBSE 2014]

39. $\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1 + \cos 4x}} dx$

40. $\int \frac{\cos ecx}{\cos ecx - \cot x} dx$

41. $\int \frac{\cos x}{1 + \cos x} dx$

42. $\int \frac{1 - \cos x}{1 + \cos x} dx$

43. $\int \sin^{-1}(\cos x) dx$

44. $\int \tan^{-1}(\sec x + \tan x) dx$

45. $\int \tan^{-1}\left(\frac{\sin 2x}{1 + \cos 2x}\right) dx$

46. $\int \sin^{-1}\left(\frac{2 \tan x}{1 + \tan^2 x}\right) dx$

47. $\int \frac{2^x + 3^x}{5^x} dx$

48. $\int \frac{(a^x + b^x)^2}{a^x b^x} dx$

49. $\int \left(3 \sin x - 4 \cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x\right) dx$

50. Write the anti derivative of $\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)$
[CBSE 2015]

Integration by Substitution

1. $\int e^{2x-3} dx$

2. $\int a^{3x+2} dx$

3. $\int \sec^2(7-4x) dx$

4. $\int \sin(ax+b) \cdot \cos(ax+b) dx$

5. $\int \sin(mx) dx$

6. $\int \frac{\sin 4x}{\cos 2x} dx$

7. $\int \sin^2(2x+5) dx$

8. $\int \sin^3(2x+1) dx$

9. $\int \cos^4 2x dx$

10. $\int \sin^3 x \cos^3 x dx$

11. $\int \sin 4x \cos 7x dx$ [CBSE 2007]

12. $\int \cos 3x \cos 4x dx$

13. $\int \cos x \cos 2x \cos 3x dx$

14. $\int \cos 2x \cos 4x \cos 6x dx$

15. $\int \sin x \sin 2x \sin 3x dx$ [CBSE 2012]

16. $\int \sin x \sqrt{1 + \cos 2x} dx$

17. $\int \frac{1}{1 - \sin \frac{x}{2}} dx$

18. $\int \frac{1}{1 + \cos 3x} dx$

19. $\int \frac{1 + \cos 4x}{\cot x - \tan x} dx$

20. $\int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} dx$

21. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

22. $\int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$

23. $\int \frac{\sec x \tan x}{3 \sec x + 5} dx$

24. $\int \frac{e^x + 1}{e^x + x} dx$

25. $\int \frac{1}{x \log x} dx$

26. $\int \frac{\sin 2x}{a \cos^2 x + b \sin^2 x} dx$

27. $\int \frac{1 - \sin x}{x + \cos x} dx$

28. $\int \frac{\sin(\tan^{-1} x)}{1 + x^2} dx$

29. $\int \frac{1}{e^x + 1} dx$

30. $\int \frac{\cos x}{2 + 3 \sin x} dx$

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| <p>31. $\int \frac{\cot x}{\log \sin x} dx$</p> <p>32. $\int \frac{1}{x+x \log x} dx$</p> <p>33. $\int \sqrt{ax+b} dx$</p> <p>34. $\int x\sqrt{x+2} dx$</p> <p>35. $\int (4x+2)\sqrt{x^2+x+1} dx$</p> <p>36. $\int \frac{1}{x-\sqrt{x}} dx$</p> <p>37. $\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx$</p> <p>38. $\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx$</p> <p>39. $\int \frac{x}{9-4x^2} dx$</p> <p>40. $\int \frac{\sec x}{\log(\sec x + \tan x)} dx$</p> <p>41. $\int \frac{1}{x \log x \log(\log x)} dx$</p> <p>42. $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$</p> <p>43. $\int \frac{1 + \tan x}{x + \log \sec x} dx$</p> <p>44. $\int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$</p> <p>45. $\int (x^3 - 1)^{\frac{1}{3}} x^5 dx$</p> <p>46. $\int \frac{\sin 2x}{\sin 5x \sin 3x} dx$</p> <p>47. $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$</p> <p>48. $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$</p> <p>49. $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$</p> <p>50. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$</p> | <p>51. $\int \sqrt{\sin 2x} \cos 2x dx$</p> <p>52. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$</p> <p>53. $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$</p> <p>54. $\int \frac{1}{\sqrt{1-x^2} (2+3 \sin^{-1} x)} dx$</p> <p>55. $\int \frac{\sec x}{\sec 2x} dx$</p> <p>56. $\int \frac{(x+1)(x+\log x)^2}{x} dx$</p> <p>57. $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$</p> <p>58. $\int \frac{1}{1-\tan x} dx$</p> <p>59. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$</p> <p>60. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$</p> <p>61. $\int \frac{\sin(x-a)}{\sin x} dx$</p> <p>62. $\int \frac{\sin x}{\sin(x-a)} dx$</p> <p>63. $\int \frac{1}{\sin(x-a) \sin(x-b)} dx$</p> <p>64. $\int \frac{1}{\sin(x-a) \cos(x-b)} dx$</p> <p>65. $\int \frac{1}{\cos(x-a) \cos(x-b)} dx$</p> <p>66. $\int \frac{\sin(x+a)}{\sin(x+b)} dx$</p> <p>67. $\int \frac{\sin 2x}{\sin\left(x-\frac{\pi}{3}\right) \sin\left(x+\frac{\pi}{3}\right)} dx$</p> <p>68. $\int \tan x \tan 2x \tan 3x dx$</p> <p>69. $\int \tan(x-\theta) \tan(x+\theta) \tan 2x dx$</p> |
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[CBSE 2009]

$$70. \int \frac{1}{\cos 3x - \cos x} dx$$

$$71. \int \frac{\tan x \sec^2 x}{(a + b \tan^2 x)^2} dx$$

$$72. \int \sec^3 x \tan x dx$$

$$73. \int x^2 \frac{\tan^{-1} x^3}{1+x^6} dx$$

$$74. \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

$$75. \int \sqrt{\tan x} (1 + \tan^2 x) dx$$

$$76. \int \frac{\sin 2x}{(a + b \cos x)^2} dx$$

$$77. \int \frac{\cos^9 x}{\sin x} dx$$

$$78. \int \frac{\cot x}{\sqrt{\sin x}} dx$$

$$79. \int \frac{\sec^4 x}{\sqrt{\tan x}} dx$$

$$80. \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx$$

$$81. \int \frac{1}{x^2 (x^4 + 1)^{\frac{3}{4}}} dx$$

$$82. \int \tan^3 x \sec^2 x dx$$

$$83. \int \tan^5 x \sec^4 x dx$$

$$84. \int \sec^6 x \tan x dx$$

$$85. \int \sec^4 2x dx$$

$$86. \int \tan^5 x dx$$

$$87. \int \cot^5 x dx$$

$$88. \int \sin^4 x \cos^3 x dx$$

$$89. \int \sin^5 x dx$$

$$90. \int \cos^5 x dx$$

$$91. \int \sin^5 x \cos x dx$$

[2014]

$$92. \int \sin^7 x dx$$

$$93. \int \frac{1}{\sin^3 x \cos^5 x} dx$$

$$94. \int \frac{\sin^4 x}{\cos^8 x} dx$$

$$95. \int \frac{1}{\sin^3 x \cos x} dx$$

$$96. \int \sec^{\frac{4}{3}} x \cos ec^{\frac{8}{3}} x dx$$

$$97. \int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$$

$$98. \int \frac{x^3 - 1}{x^3 + x} dx$$

[CBSE 2015]

Some Standard Form of Integrals

$$1. \int \frac{1}{4+9x^2} dx$$

$$2. \int \frac{1}{9x^2-4} dx$$

$$3. \int \frac{1}{\sqrt{9-25x^2}} dx$$

$$4. \int \frac{1}{\sqrt{16x^2+25}} dx$$

$$5. \int \frac{1}{a^2-b^2x^2} dx$$

$$6. \int \frac{1}{a^2x^2+b^2} dx$$

$$7. \int \frac{x^2-1}{x^2+4} dx$$

$$8. \int \frac{1}{\sqrt{a^2-b^2x^2}} dx$$

$$9. \int \frac{1}{\sqrt{(2-x)^2+1}} dx$$

$$10. \int \frac{x^4}{x^2+1} dx$$

$$11. \int \frac{1}{x^2-x+1} dx$$

12. $\int \frac{1}{2x^2 + x - 1} dx$
13. $\int \frac{1}{3x^2 + 13x - 10} dx$
14. $\int \frac{1}{9x^2 + 6x + 10} dx$
15. $\int (x-3)\sqrt{x^2 + 3x - 18} dx$ [2014]
16. $\int (3x-2)\sqrt{x^2 + x + 1} dx$ [2014]
17. $\int \frac{1}{x^2 + 6x + 13} dx$
18. $\int \frac{1}{1+x-x^2} dx$
19. $\int \frac{1}{x(x^5+1)} dx$
20. $\int \frac{\sec^2 x}{1-\tan^2 x} dx$
21. $\int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$
22. $\int \frac{e^x}{e^{2x} + 5e^x + 6} dx$
23. $\int \frac{e^{3x}}{4e^{6x} - 9} dx$
24. $\int \frac{1}{e^x + e^{-x}} dx$
25. $\int \frac{x}{x^4 + 2x^2 + 3} dx$
26. $\int \frac{3x^5}{1+x^{12}} dx$
27. $\int \frac{x^2}{x^6 - a^6} dx$
28. $\int \frac{x}{x^4 - x^2 + 1} dx$ [CBSE 2007]
29. $\int \frac{1}{\sqrt{(x-1)(x-2)}} dx$
30. $\int \frac{1}{\sqrt{9+8x-x^2}} dx$
31. $\int \frac{1}{\sqrt{2x^2+3x-2}} dx$
32. $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$
33. $\int \frac{1}{\sqrt{5-4x-2x^2}} dx$ [CBSE 2009]
34. $\int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$
35. $\int \frac{1}{\sqrt{5x^2-2x}} dx$
36. $\int \frac{1}{\sqrt{7-6x-x^2}} dx$
37. $\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}} dx$
38. $\int \frac{1}{x\sqrt{(\log x)^2-5}} dx$
39. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$ [CBSE 2009]
40. $\int \sqrt{\frac{x}{a^3-x^3}} dx$
41. $\int \frac{\sin 2x \cos 2x}{\sqrt{9-\cos^4 2x}} dx$
42. $\int \frac{\sin 2x}{\sqrt{\sin^4 x + 4 \sin^2 x - 2}} dx$
43. $\int \frac{1}{\sqrt{(1-x^2)\{9+(\sin^{-1} x)^2\}}} dx$
44. $\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx$ [CBSE 2011]
45. $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx$
46. $\int \frac{1}{\sqrt{1-e^{2x}}} dx$
47. $\int \sqrt{\sec x - 1} dx$
48. $\int \frac{x}{x^2+x+1} dx$
49. $\int \frac{4x+1}{x^2+3x+2} dx$

$$50. \int \frac{2x-3}{x^2+3x-18} dx$$

$$51. \int \frac{x^3+x}{x^4-9} dx$$

$$52. \int \frac{2x-3}{x^2+6x+13} dx$$

$$53. \int \frac{2x}{2+x-x^2} dx$$

$$54. \int \frac{1-3x}{3x^2+4x+2} dx$$

$$55. \int \frac{ax^3+bx}{x^4+c^2} dx$$

$$56. \int \frac{x+2}{2x^2+6x+5} dx \quad \text{[CBSE 2007]}$$

$$57. \int \frac{5x-2}{1+2x+3x^2} dx \quad \text{[CBSE 2013]}$$

$$58. \int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x} dx \quad \text{[CBSE 2013]}$$

$$59. \int \frac{x^2+5x+3}{x^2+3x+2} dx$$

$$60. \int \frac{1-x^2}{x(1-2x)} dx \quad \text{[CBSE 2010]}$$

$$61. \int \frac{(x-1)^2}{x^2+2x+2} dx$$

$$62. \int \frac{x^2(x^4+4)}{x^2+4} dx$$

$$63. \int \frac{x+2}{\sqrt{x^2+5x+6}} dx \quad \text{[CBSE 2010]}$$

$$64. \int \sqrt{\frac{1+x}{x}} dx$$

$$65. \int \frac{x+2}{\sqrt{x^2-1}} dx$$

$$66. \int \frac{x+1}{\sqrt{x^2+1}} dx$$

$$67. \int \sqrt{\frac{1-x}{1+x}} dx$$

$$68. \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx \quad \text{[CBSE 2011, 12]}$$

$$69. \int \frac{x+2}{\sqrt{x^2+2x+3}} dx \quad \text{[CBSE 2013]}$$

$$70. \int \sqrt{\frac{a-x}{a+x}} dx$$

$$71. \int x \sqrt{\frac{a^2-x^2}{a^2+x^2}} dx$$

$$72. \int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

$$73. \int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$$

$$74. \int \frac{1}{(2\sin x+3\cos x)^2} dx$$

$$75. \int \frac{1}{3+\sin 2x} dx$$

$$76. \int \frac{1}{2-3\cos 2x} dx$$

$$77. \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

$$78. \int \frac{1}{\cos x(\sin x+2\cos x)} dx$$

$$79. \int \frac{1}{\sin^2 x + \sin 2x} dx$$

$$80. \int \frac{1}{\cos 2x + 3\sin^2 x} dx$$

$$81. \int \frac{\cos x}{\cos 3x} dx$$

$$82. \int \frac{\sin x}{\sin 3x} dx$$

$$83. \int \frac{1}{1+\sin x+\cos x} dx$$

$$84. \int \frac{1}{2+\cos x} dx$$

$$85. \int \frac{1}{1-2\sin x} dx$$

$$86. \int \frac{1+\sin x}{\sin x(1+\cos x)} dx$$

$$87. \int \frac{1}{3+2\sin x+\cos x} dx$$

$$88. \int \frac{1}{\sqrt{3}\sin x+\cos x} dx$$

$$89. \int \frac{1}{\sin x + \sqrt{3} \cos x} dx$$

$$90. \int \frac{1}{\sin x - \sqrt{3} \cos x} dx$$

$$91. \int \frac{1}{13 + 3 \cos x + 4 \sin x} dx$$

$$92. \int \frac{3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx$$

$$93. \int \frac{4 \sin x + 5 \cos x}{5 \sin x + 4 \cos x} dx$$

$$94. \int \frac{1}{1 + \cot x} dx$$

$$95. \int \frac{1}{4 + 3 \tan x} dx$$

$$96. \int \frac{2 \tan x + 3}{3 \tan x + 4} dx$$

$$97. \int \frac{8 \cot x + 1}{3 \cot x + 2} dx$$

$$98. \int \frac{5 \cos x + 6}{2 \cos x + \sin x + 3} dx$$

Integration By Parts

$$1. \int x \sec^2 x dx$$

$$2. \int x \log x dx$$

$$3. \int x \sin 3x dx$$

$$4. \int x^2 e^x dx$$

$$5. \int x \sin^{-1} x dx \quad \text{[CBSE 2009, 2015]}$$

$$6. \int x \tan^{-1} x dx$$

$$7. \int (\sin^{-1} x)^2 dx$$

$$8. \int \tan^{-1} x dx$$

$$9. \int \log x dx$$

$$10. \int x (\log x)^2 dx$$

$$11. \int \sec^{-1} x dx$$

$$12. \int \frac{\log x}{x^2} dx$$

$$13. \int \log(1+x^2) dx$$

$$14. \int \frac{x - \sin x}{1 - \cos x} dx$$

$$15. \int (x^2 + 1) \log x dx$$

$$16. \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

$$17. \int x^3 \log 2x dx$$

$$18. \int x^2 \sin^2 x dx$$

$$19. \int x^n \log x dx$$

$$20. \int \sin x \log(\cos x) dx$$

$$21. \int \cos \sqrt{x} dx$$

$$22. \int x \cos^3 x dx$$

$$23. \int x \tan^2 x dx$$

$$24. \int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

$$25. \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx \quad \text{[CBSE 2012]}$$

$$26. \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$$

$$27. \int \frac{\sin^{-1} x}{x^2} dx$$

$$28. \int x^2 \tan^{-1} x dx \quad \text{[CBSE 2012]}$$

$$29. \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$30. \int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

$$31. \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$$

$$32. \int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx \quad \text{[2012]}$$

$$33. \int \log_{10} x dx$$

$$34. \int e^x (\sin x + \cos x) dx$$

$$35. \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$36. \int [\sin(\log x) + \cos(\log x)] dx$$

$$37. \int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) dx$$

$$38. \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$39. \int \frac{(x-3)e^x}{(x-1)^3} dx$$

$$40. \int \frac{(x+3)e^x}{(x+5)^3} dx \quad [2015]$$

$$41. \int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$$

$$42. \int \frac{\log x}{(1 + \log x)^2} dx$$

$$43. \int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

$$44. \int e^x \frac{(1-x)^2}{(1+x^2)^2} dx$$

$$45. \int e^{2x} \left(\frac{1 + \sin 2x}{1 + \cos 2x} \right) dx \quad [CBSE 2010]$$

$$46. \int \frac{e^x}{x} \left\{ x(\log x)^2 + 2 \log x \right\} dx$$

$$47. \int e^x \frac{(x-4)}{(x-2)^3} dx \quad [CBSE 2009]$$

$$48. \int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx \quad [CBSE 2013]$$

$$49. \int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} dx \quad [CBSE 2013]$$

$$50. \int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$$

[CBSE 2010]

$$51. \int \{ \tan(\log x) + \sec^2(\log x) \} dx$$

$$52. \int e^{2x} \sin 3x dx$$

$$53. \int \sin(\log x) dx$$

$$54. \int e^{2x} \sin x dx$$

$$55. \int e^x \sin^2 x dx$$

$$56. \int \frac{1}{x^3} \sin(\log x) dx$$

$$57. \int e^{-2x} \sin x dx$$

Integrals of some special form

$$1. \int \sqrt{x^2 + 2x + 5} dx$$

$$2. \int \sqrt{7x - 10 - x^2} dx$$

$$3. \int \sqrt{(x-3)(5-x)} dx$$

$$4. \int \sqrt{3 + 2x - x^2} dx$$

$$5. \int \sqrt{x - x^2} dx$$

$$6. \int \sqrt{2ax - x^2} dx$$

$$7. \int \sqrt{3 - 2x - 2x^2} dx$$

$$8. \int \sqrt{1 + \frac{x^2}{9}} dx$$

$$9. \int x^2 \sqrt{a^6 - x^6} dx$$

$$10. \int \frac{\sqrt{16 + (\log x)^2}}{x} dx$$

$$11. \int (x+1)\sqrt{1-x-x^2} dx$$

$$12. \int x\sqrt{x^2 + x} dx$$

$$13. \int (2x+3)\sqrt{x^2 + 4x + 3} dx$$

$$14. \int (2x-5)\sqrt{x^2 - 4x + 3} dx$$

$$15. \int x\sqrt{1+x-x^2} dx$$

Integration using Partial Fractions

1. $\int \frac{x}{(x+1)(x+2)} dx$
2. $\int \frac{1}{x^2-9} dx$
3. $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$
4. $\int \frac{2x}{x^2+3x+2} dx$
5. $\int \frac{1-x^2}{x(1-2x)} dx$
6. $\int \frac{x}{(x^2+1)(x-1)} dx$
7. $\int \frac{x}{(x-1)^2(x+2)} dx$
8. $\int \frac{3x+5}{x^3-x^2-x+1} dx$
9. $\int \frac{x^3}{(x-1)(x-2)} dx$
10. $\int \frac{2x}{(x^2+1)(x^2+2)} dx$ [CBSE 2011]
11. $\int \frac{x^3+x+1}{x^2-1} dx$
12. $\int \frac{1}{\sin x - \sin 2x} dx$ [CBSE 2010]
13. $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$ [CBSE 2012]
14. $\int \frac{8}{(x+2)(x^2+4)} dx$ [CBSE 2013]
15. $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$ [CBSE 2013]
16. $\int \frac{x^2+1}{(x^2+2)(2x^2+1)} dx$
17. $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$
18. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$ [CBSE 2013]
19. $\int \frac{x}{(x^2-a^2)(x^2-b^2)} dx$
20. $\int \frac{1}{x(x^3+8)} dx$ [CBSE 2013]
21. $\int \frac{3}{(1-x)(1+x^2)} dx$ [CBSE 2012]
22. $\int \frac{1}{x^4-1} dx$
23. $\int \frac{x^4}{(x-1)(x^2+1)} dx$
24. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$
25. $\int \frac{1}{e^x-1} dx$
26. $\int \frac{\cos x}{(1-\sin x)^3(2+\sin x)} dx$
27. $\int \frac{1}{x(x^4-1)} dx$
28. $\int \frac{1}{x(x^n+1)} dx$
29. $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$
30. $\int \frac{1-\cos x}{\cos x(1+\cos x)} dx$ [CBSE 2015]
31. $\int \frac{ax^2+bx+c}{(x-a)(x-b)(x-c)} dx$
32. $\int \frac{\sin x}{\sin 4x} dx$
33. $\int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} d\theta$

Miscellaneous Exercise

1. $\int \frac{x^2+1}{x^4+1} dx$ [CBSE 2007, 11]

2. $\int \frac{x^2+4}{x^4+16} dx$ [CBSE 2007]

3. $\int \frac{x^2-1}{x^4+x^2+1} dx$

4. $\int \frac{(x-1)^2}{x^4+x^2+1} dx$

5. $\int \frac{x^2-3x+1}{x^4+x^2+1} dx$

6. $\int \frac{x^2}{x^4+x^2-2} dx$ [2015]

7. $\int \sqrt{\tan \theta} d\theta$

8. $\int (\sqrt{\tan \theta} + \sqrt{\cot \theta}) d\theta$ [CBSE 2015]

9. $\int \frac{1}{\sin^4 x + \cos^4 x} dx$

10. $\int \frac{1}{(x-1)\sqrt{2x+3}} dx$

11. $\int \frac{x+1}{(x-1)\sqrt{x+2}} dx$

12. $\int \frac{x^2}{(x-1)\sqrt{x+2}} dx$

13. $\int \frac{1}{(x^2+1)\sqrt{x}} dx$

14.

15. $\int \frac{x}{(x^2+2x+2)\sqrt{x+1}} dx$

16. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$

17. $\int \frac{1}{(x+1)\sqrt{x^2-1}} dx$

18. $\int \frac{1}{(x-1)\sqrt{x^2+4}} dx$

19. $\int \frac{1}{x^2\sqrt{1+x^2}} dx$

20. $\int \frac{1}{(x^2-1)\sqrt{x^2+1}} dx$

21. $\int \frac{\sqrt{1+x^2}}{1-x^2} dx$

22. $\int \frac{x}{(x^2+4)\sqrt{x^2+9}} dx$

23. $\int \cos 6x\sqrt{1+\sin 6x} dx$

24. $\int \frac{(x^4-x)^{\frac{1}{4}}}{x^5} dx$

25. $\int \frac{x^4}{(x-1)(x^2+1)} dx$

26. $\int \frac{\sin 2x \cos 2x}{\sqrt{9-\cos^4(2x)}} dx$

27. $\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx$

28. $\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} dx$

29. $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x \cos^2 x} dx$

30. $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$

31. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

32. $\int \frac{1}{e^x + e^{-x}} dx$

7B. DEFINITE INTEGRALS

Evaluate the following integrals:

1. $\int_2^3 x^2 dx$

2. $\int_4^9 \frac{\sqrt{x}}{(30-x^{\frac{3}{2}})^2} dx$

3. $\int_1^2 \frac{x}{(x+1)(x+2)} dx$

4. $\int_0^{\frac{\pi}{4}} \sin^3 2t \cos 2t dt$

5. $\int_{-1}^1 (x+1) dx$

6. $\int_2^3 \frac{1}{x} dx$

7. $\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$

8. $\int_0^{\frac{\pi}{4}} \sin 2x dx$

9. $\int_4^5 e^x dx$

10. $\int_0^{\frac{\pi}{2}} \cos 2x dx$

11. $\int_0^{\frac{\pi}{4}} \tan x dx$

[CBSE 2014]

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x dx$

13. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

14. $\int_0^1 \frac{dx}{1+x^2}$

15. $\int_0^3 \frac{dx}{9+x^2}$

[2014]

16. $\int_2^3 \frac{dx}{x^2-1}$

17. $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

[CBSE 2002]

18. $\int_2^3 \frac{x dx}{x^2+1}$

19. $\int_0^1 \frac{2x+3}{5x^2+1} dx$

20. $\int_0^1 x e^{x^2} dx$

21. $\int_1^2 \frac{5x^2}{x^2+4x+3} dx$

[CBSE 2010]

22. $\int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx$

23. $\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$

24. $\int_0^2 \frac{6x+3}{x^2+4} dx$

25. $\int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$

26. $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin 2x} dx$

[CBSE 2004]

27. $\int_1^3 \frac{1}{x^2(x+1)} dx$

28. $\int_0^2 \frac{1}{4+x-x^2} dx$

29. $\int_1^2 \left(\frac{x-1}{x^2} \right) e^x dx$

[CBSE 2000, 2002]

30. $\int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$

31. $\int_{-1}^1 \frac{1}{x^2+2x+5} dx$

32. $\int_0^{\frac{\pi}{2}} \sin^3 x dx$

33. $\int_1^2 e^{2x} \left(\frac{1}{x} - \frac{1}{2x^2} \right) dx$

34. $\int_0^1 \frac{1}{\sqrt{1+x}-\sqrt{x}} dx$

35. $\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

36. $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$

37. $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

38. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9+16\sin 2x} dx$

39. $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$

40. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log \sin x dx$ [CBSE 2003]

41. Evaluate $\int_{-1}^1 5x^4 \sqrt{x^5+1} dx$

42. Evaluate $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

43. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \cos ecx} dx$ [CBSE 2015]

Evaluate the following integrals using substitution method

44. $\int_0^1 \frac{x}{x^2+1} dx$

45. $\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$

46. $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$ [CBSE 2002]

47. $\int_0^2 x \sqrt{x+2} dx$

48. $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

49. $\int_0^2 \frac{dx}{x+4-x^2}$

50. $\int_{-1}^1 \frac{dx}{x^2+2x+5}$

51. $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$

52. $\int_0^{\pi/2} \frac{\cos \theta}{(1+\sin \theta)(2+\sin \theta)} d\theta$ [CBSE 2004]

53. $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$ [CBSE 2007]

54. $\int_0^{\pi/4} \tan^3 x dx$ [CBSE 2004]

55. $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$ [CBSE 2002, 2003]

56. $\int_0^{\pi} \frac{1}{5+4\cos x} dx$ [CBSE 2005]

57. $\int_0^{\pi/2} \frac{1}{2\cos x + 4\sin x} dx$

58. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$ [CBSE 2007, 2011]

59. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ [CBSE 2014]

60. $\int_0^{\pi/2} \frac{1}{5\cos x + 3\sin x} dx$

61. $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ [CBSE 2004]

62. $\int_0^{\pi/2} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

63. $\int_1^2 \frac{1}{x(1+\log x)^2} dx$ [CBSE 2003]

64. $\int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x) dx$ [CBSE 2011]

65. $\int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x}) dx$ [CBSE 2012]

66. $\int_0^{\pi/4} \sin^3 2t \cos 2t dt$

67. $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$ [CBSE 2011]

68. Evaluate $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ [CBSE 2003C]

69. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

70. Evaluate $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$ [CBSE 2012]

Evaluate the following integrals using properties of definite integrals

71. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

72. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

73. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

74. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

75. Evaluate $\int_0^{\pi/2} \log \tan x dx$ [CBSE 2007]

Evaluate:

76. $\int_0^{\pi/4} \log(1+\tan x) dx$

[CBSE 2002C, 2003, 2004, 2011]

77. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

78. $\int_0^{\pi/2} (2\log \sin x - \log \sin 2x) dx$ [CBSE 2009]

79. $\int_0^1 x(1-x)^n dx$

80. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$ [CBSE 2002, 2003, 2015]

81. $\int_0^1 \cot^{-1}(1-x+x^2) dx$ [CBSE 2008]

82. $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

[CBSE 2003, 2005, 2008, 2011, 2012]

83. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ [CBSE 2008, 2010, 2014]

84. $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ [CBSE 2010, 2011]

85. $\int_0^{\pi/2} \cos^2 x dx$

86. $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$

87. $\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$

88. $\int_0^2 x\sqrt{2-x} dx$ [CBSE 2007]

89. $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$ [CBSE 2004]

90. $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ [CBSE 2015]

91. $\int_0^{\pi} \log(1 + \cos x) dx$

92. $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

93. Prove that $\int_0^{\pi/2} \sin 2x \log \tan x dx = 0$.

94. Prove that $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$

95. Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$.

Evaluate

96. $\int_{-5}^5 |x+2|$

97. $\int_2^8 |x-5| dx$

98. $\int_0^4 |x-1| dx$

99. $\int_{-1}^2 |x^3 - x| dx$ [CBSE 2012]

100. $\int_{1/e}^e |\log_e x|$

101. $\int_0^{2\pi} |\sin x|$

102. $\int_0^{\pi/2} |\cos 2x|$

103. $\int_{-1}^{3/2} |x \sin(\pi x)| dx$

104. $\int_0^{3/2} |x \cdot \cos(\pi x)| dx$ [CBSE 2015]

105. $\int_1^4 [|x-1| + |x-2| + |x-3|] dx$

106. $\int_{-\pi/2}^{\pi/2} \{\sin|x| + \cos|x|\} dx$ [CBSE 2000]

107. $\int_{-5}^0 f(x) dx$, where $f(x) = |x| + |x+2| + |x+5|$ [CBSE 2005]

108. $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$ [CBSE 2011]

109. $\int_{-\pi/2}^{\pi/2} \sin^2 x dx$

110. $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$

111. $\int_0^{2\pi} \cos^5 x dx$

112. Evaluate $\int_{-\pi/4}^{\pi/4} \sin^2 x dx$

113. Evaluate $\int_{-1}^1 \sin^5 x \cos^5 x dx$

Evaluate

114. $\int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx$ [CBSE 2002, 2008]

115. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ [CBSE 2008]

116. Prove that $\int_0^{2a} f(x) dx = \int_0^{2a} f(2a-x) dx$. [CBSE 2002C]

117. Evaluate $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$ [CBSE 2008]

118. Evaluate $\int_0^{\pi} \frac{x}{1 + \sin x} dx$ [CBSE 2001C, 2004, 2001, 2010, 2012]

119. Prove that $\int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log 2$

Prove the following:

120. $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$

121. $\int_0^1 x e^x dx = 1$

122. $\int_{-1}^1 x^{17} \cos^4 x dx = 0$

123. $\int_0^{\frac{\pi}{2}} \sin^3 x dx = \frac{2}{3}$

124. $\int_0^{\frac{\pi}{4}} \tan^3 x dx = 1 - \log 2$

125. $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$

Evaluate:

126. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

127. $\int_0^{2/3} \frac{dx}{4+9x^2}$

128. $\int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$

129. Find the value of integral $\int_{1/3}^1 \frac{(x-x^3)^{1/3}}{x^4} dx$.

130. Find the value of $f'(x)$, if $f(x) = \int_0^x t \sin t dt$.

131. Find the value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$

132. Find the value of $\int_0^{\pi/2} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$.

LIMIT OF A SUM

133. Find $\int_0^2 (x^2+1) dx$ as the limit of a sum.

134. Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

Evaluate the following definite integrals as limit of sum

135. $\int_a^b x dx$

136. $\int_0^5 (x+1) dx$

137. $\int_2^3 x^2 dx$

138. $\int_1^4 (x^2 - x) dx$ [CBSE 2010, 2012]

139. $\int_{-1}^1 e^x dx$

140. $\int_0^4 (x + e^{2x}) dx$

141. $\int_0^2 (x^2 + 3) dx$ [CBSE 2001 C]

142. $\int_1^3 (2x^2+5) dx$ [CBSE 2010]

143. $\int_1^3 (x^2 + x) dx$ [CBSE 2000 C]

144. $\int_1^3 (x^2 + 5x) dx$ [CBSE 2010]

145. $\int_0^1 e^{2-3x} dx$

146. $\int_0^2 (x^2 + 2x + 1) dx$ [CBSE 2007]

147. $\int_0^2 (x^2 - x) dx$ [CBSE 2011]

148. $\int_0^2 (x^2 + 2) dx$ [CBSE 2000]

149. $\int_0^2 (x^2 + x) dx$ [CBSE 2005]

150. $\int_0^3 (2x^2 + 3x + 5) dx$ [CBSE 2007]

151. $\int_1^3 (x^2 + x) dx$ [CBSE 2012]

152. $\int_1^3 (2x^2 + 5x) dx$ [CBSE 2012]

153. Evaluate $\int_{-1}^2 (e^{3x} + 7x - 5)$ as a limit of sums. [2015]

154. Find $\int_0^2 (x^2 + e^{2x+1}) dx$ as the limit of a sum. [2015]