2. INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the principal values of each of the following:

(i) $\tan^{-1}\left(-\sqrt{3}\right)$ **[CBSE 2011]**

(ii) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(iii) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(iv) $\cot^{-1}\left(-\sqrt{3}\right)$

(v) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

(vi) $\sin^{-1}\left(-\frac{1}{2}\right)$ **[CBSE 2011]**

(vii) $\sec^{-1}\left(-\sqrt{2}\right)$

(viii) $\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

2. Evaluate each of the following:

(i) $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$ **[CBSE 2012, 2014]**

(ii) $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$ **[CBSE 2012]**

(iii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[CBSE 2008, CBSE 2011]**

(iv) $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}\left(-2\right)$ **[CBSE 2012]**

(v) $\tan^{-1}\left(-1\right)+\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(vi) $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}\left(-2\right)+\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(vii) $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\sqrt{3}\right)$ **[CBSE 2013]**

3. Evaluate each of the following:

(i) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

(ii) $\cos^{-1}\left\{\cos\left(-\frac{\pi}{2}\right)\right\}$

(iii) $\tan^{-1}\left\{\tan\left(\frac{3\pi}{4}\right)\right\}$ **[CBSE 2009, 2011]**

(iv) $\cos^{-1}\left\{\cos\left(\frac{5\pi}{2}\right)\right\}$

(v) $\sin^{-1}\left(\sin 2\right)$

(vi) $\tan^{-1}\left\{\tan\left(\frac{2\pi}{3}\right)\right\}$

(vii) $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

(viii) $\cos\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{4}\right\}$

(ix) $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$

(x) $\sin\left\{\arccos\left(-\frac{1}{2}\right)\right\}$

(xi) $\sin\left\{\tan^{-1}\left(-\sqrt{3}\right)+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

(xii) $\sin\left\{\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

(xiii) $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$ **[CBSE 2008, 2011]**

4. Evaluate each of the following:

(i) $\cos\left(\sin^{-1}\frac{3}{5}\right)$

(ii) $\sin\left(\cos^{-1}\frac{4}{5}\right)$

(iii) $\tan\left(\cos^{-1}\frac{8}{17}\right)$

(iv) $\cos\left\{\sin^{-1}\left(-\frac{3}{5}\right)\right\}$

(v) $\operatorname{cosec}\left\{\cos^{-1}\left(-\frac{12}{13}\right)\right\}$

(vi) $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right\}$

(vii) $\tan\left[\frac{1}{2}\left\{\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}\right]$

(viii) $\sin\left[\frac{1}{2}\left\{\cos^{-1}\left(\frac{4}{5}\right)\right\}\right]$

(ix) $\sec\left\{\tan^{-1}\left(\frac{y}{2}\right)\right\}$

(x) $\tan^{-1}\left\{2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

5. Prove the following:

(i) $\cos^{-1}\left(\frac{5}{13}\right)=\tan^{-1}\left(\frac{12}{5}\right)$

(ii) $\tan^{-1}\left(\frac{2}{11}\right)+\tan^{-1}\left(\frac{7}{24}\right)=\tan^{-1}\left(\frac{1}{2}\right)$

(iii) $\sin^{-1}\left(-\frac{4}{5}\right)=\tan^{-1}\left(-\frac{4}{3}\right)$

(iv) $2\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{24}{7}\right)$

(v) $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{2}{9}\right)$

(vi) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

(vii) $\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{63}{16}\right)$

(viii) $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$

(ix) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{17}{6}$

(x) $\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$

(xi) $2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{4}\right)=\tan^{-1}\left(\frac{32}{43}\right)$

(xii) $2\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{17}{31}\right)=\frac{\pi}{4}$

(xiii) $2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\tan^{-1}\left(\frac{4}{7}\right)$

(xiv) $\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{63}{65}\right)$

(xv) $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{77}{85}\right)$

(xvi) $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$

(xvii) $\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

(xviii) $2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)=\frac{\pi}{4}$ **[2015]**

(xix) $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

(xx) $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

(xxi) $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

[2014]

(xxii) $4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{70}\right)+\tan^{-1}\left(\frac{1}{99}\right)=\frac{\pi}{4}$

(xxiii) $\cot^{-1}(7)+\cot^{-1}(8)+\cot^{-1}(18)=\cot^{-1}(3)$

[2014]

(xxiv) $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$

(xxv) $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)\right\}=\frac{4-\sqrt{7}}{3}$

6. Write each of the following in the simplest form:

(i) $\tan^{-1}\left\{x+\sqrt{1+x^2}\right\}$

(ii) $\cot^{-1}\left\{\frac{a}{\sqrt{x^2-a^2}}\right\}$

(iii) $\tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}$

(iv) $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$

(v) $\tan^{-1}\left\{\sqrt{\frac{a-x}{a+x}}\right\}$

(vi) $\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$

(vii) $\sin^{-1}\left\{x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right\}$

(viii) $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$

(ix) $\tan^{-1}\left\{\frac{3a^2x-x^3}{a^3-3ax^2}\right\}$

(x) $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

(xi) $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

(xii) $\tan\left\{\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]\right\}$

(xiii) $\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

(xiv) $\tan^{-1}\left\{\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right\}$

(xv) $\tan^{-1}\left\{\frac{\cos x}{1+\sin x}\right\}$

7. Simplify the following:

(i) $\cos\left(\sec^{-1}x+\mathrm{cosec}^{-1}x\right)$

(ii) $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

(iii) $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

(iv) $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right\}$

(v) $\cos^{-1}\left\{\frac{3}{5}\cos x+\frac{4}{5}\sin x\right\}$

8. Prove that: $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\sqrt{\frac{x^2+1}{x^2+2}}$

9. Prove that:

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}$$

10. If $\sin\left(\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}(x)\right)=1$, then find the value of x. **[2014]**

11. Prove that: $\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{2}$

12. Prove that: $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

13. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then prove that $x=\frac{a-b}{1+ab}$.

14. Prove that: $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

15. Prove that:

$$\cot^{-1}\left\{\frac{ab+1}{a-b}\right\}+\cot^{-1}\left\{\frac{bc+1}{b-c}\right\}+\cot^{-1}\left\{\frac{ca+1}{c-a}\right\}=0$$

16. If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$, then prove that:

$$9x^2-12xy\cos\theta+4y^2=36\sin^2\theta.$$

17. Prove that

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2};\ x\in\left(0,\frac{\pi}{4}\right)$$

[2014]

18. Prove that:

$$2\tan^{-1}\left\{\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right)$$

19. Prove that:

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$$

[2015]

20. If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$, then prove that $x^2=\sin 2\alpha$.

21. Solve the following for x:

(i) $\tan^{-1}(3x)+\tan^{-1}(2x)=\frac{\pi}{4}$

(ii) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

(iii) $\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}(3x)$

(iv) $\sin^{-1}(x)+\sin^{-1}(2x)=\frac{\pi}{3}$

(v) $\cos^{-1}(x)+\sin^{-1}\left(\frac{x}{2}\right)=\frac{\pi}{6}$

(vi) $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\frac{1}{2}\tan^{-1}(x)=0$

(vii) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{2\pi}{3}$

(viii) $\cot^{-1}(x)-\cot^{-1}(x+2)=\frac{\pi}{12}$

(ix) $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$

(x) $\sin^{-1}(1-x)-2\sin^{-1}(x)=\frac{\pi}{2}$ **[2015]**

(xi) $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$ **[2014]**

(xii) $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$

(xiii) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

22. Solve the following for x:

$$\tan^{-1}\left(\frac{x-2}{x-3}\right)+\tan^{-1}\left(\frac{x+2}{x+3}\right)=\frac{\pi}{4}, |x|<1.$$

[2015]

23. If

$$\tan^{-1}(x)+\tan^{-1}(y)+\tan^{-1}(z)=\frac{\pi}{2}, x, y, z>0,$$

then find the value of $xy+yz+zx$. **[2015]**

24. Prove that:

(i) $2\tan^{-1}(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(ii) $\sin^{-1}(x)=\cos^{-1}\left(\sqrt{1-x^2}\right)$

(iii) $\sin^{-1}(x)=\cos^{-1}\left(\sqrt{1-x^2}\right)$

(iv) $\cos^{-1}(x)=2\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$

(v) $2\tan^{-1}(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

(vi) $3\sin^{-1}(x)=\sin^{-1}\left(3x-4x^3\right)$

(vii) $\tan^{-1}(x)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

25. Which is greater tan (1) or $\tan^{-1}(1)$?

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