

12. LINEAR PROGRAMMING

Solve the following linear programming problem graphically:

- Maximise : $Z = 4x + y$
Subject to $x + y \leq 50$
 $3x + y \leq 90$
 $x \geq 0, y \geq 0$.
- Minimise: $Z = 200x + 500y$
Subject to $x + 2y \geq 10$
 $3x + 4y \leq 24$
 $x \geq 0, y \geq 0$.
- Maximise: $Z = 3x + 4y$
Subject to $x + y \leq 4, x \geq 0, y \geq 0$.
- Maximise: $Z = 5x + 3y$
Subject to $3x + 5y \leq 15, 5x + 2y \leq 10,$
 $x \geq 0, y \geq 0$
- Maximise: $Z = 3x + 2y$
Subject to
 $x + 2y \leq 10$
 $3x + y \leq 15$
 $x, y \geq 0$
- Minimise: $Z = x - 5y + 20$
Subject to
 $x - y \geq 0$
 $-x + 2y \geq 2$
 $x \geq 3, y \leq 4$
 $x, y \geq 0$
- Minimise: $Z = 3x_1 + 5x_2$
Subject to
 $x_1 + 3x_2 \geq 3$
 $x_1 + x_2 \geq 2$
 $x_1, x_2 \geq 0$
- Maximise: $Z = 60x + 15y$
Subject to
 $x + y \leq 50$
 $3x + y \leq 90$
 $x, y \geq 0$
- Minimise: $Z = -50x + 20y$
Subject to

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, y \geq 0$$

- Minimise: $Z = 3x + 2y$
Subject to
 $x + y \geq 8$
 $3x + 5y \leq 15$
 $x \geq 0, y \geq 0$
- Minimise: $Z = -3x + 4y$
Subject to
 $x + y \leq 8$
 $3x + 2y \leq 12$
 $x \geq 0, y \geq 0$
- Minimise: $Z = 3x + 5y$
Subject to
 $x + 3y \geq 3$
 $x + y \geq 2$
 $x, y \geq 0$
- Minimise: $Z = x + 2y$
Subject to
 $2x + y \geq 3$
 $x + 2y \geq 6$
 $x, y \geq 0$

Show that the minimum of Z occurs at more than two points.

- Minimise and Maximise: $Z = 5x + 10y$
Subject to
 $x + 2y \leq 120$
 $x + y \geq 60$
 $x - 2y \geq 0$
 $x, y \geq 0$
- Minimise and Maximise: $Z = x + 2y$
Subject to
 $x + 2y \geq 100$
 $2x - y \leq 0$
 $2x + y \leq 200$
 $x, y \geq 0$

- Minimize and maximize $z = 5x + 2y$ subject to the following constraints:

[CBSE 2015]

$$\begin{aligned}x - 2y &\leq 2 \\3x + 2y &\leq 12 \\-3x + 2y &\leq 3 \\x \geq 0, y &\geq 0\end{aligned}$$

17. Maximise: $Z = -x + 2y$

Subject to

$$\begin{aligned}x &\geq 3 \\x + y &\geq 5 \\x + 2y &\geq 6 \\y &\geq 0\end{aligned}$$

18. Maximise: $Z = x + y$

Subject to

$$\begin{aligned}x - y &\leq -1 \\-x + y &\leq 0 \\x, y &\geq 0\end{aligned}$$

19. A manufacturing company takes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing the maximum labour hours available are 180, 30 respectively. The company makes a profit of ₹8000 on each piece of model A and ₹12,000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realise a maximum profit? What is maximum profit per week?

20. A factory makes tennis rackets and cricket bats. A tennis rackets takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time.

(a) What number of rackets and bats must be made if the factory is to work at full capacity?

(b) If the profit on a racket and on a bat is ₹20 and ₹10 respectively. Find the

maximum profit of the factory when it works at full capacity?

21. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

22. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on hand operated machines to manufacture a package of screws B. Each machine is available for at most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of ₹7 and screws B at a profit of ₹10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit?

23. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/ cutting machine and a sprayer. It takes 2 hours on grinding/ cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the cutting/ grinding machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/ cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹5 and that from a shade is ₹3. Assuming that the manufacturer can sell all the lamps and

shades that he produces, how should he schedule his daily production in order to maximise his profit? **[CBSE 2014]**

24. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B requires 8 minute each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is `5 each for type A and `6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?
25. A manufacturer has 3 machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of `600 and `400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

26. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minute) required for each toy on the machine is given below:

Types of toys	Machines		
	I	II	III
A	12	18	6

B	6	0	9
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- Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is `7.50 and that on each toy of type B is `5, show that 15 toys of type A and 30 toys of type B should be manufactured in a day to get maximum profit.
27. A toy company manufactures two types of dolls A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of `12 and `16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?
28. Two tailors A and B earn `150 and `200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants. **[CBSE 2005]**
29. A manufacturer makes two types A and B of tea - cups. Three machines are needed for the manufacture and the time(in minutes) required for each cup on the machines is given below:

	Machines		
	I	II	III
A	12	18	06
B	06	00	09

- Each machine is available for a maximum of 6 hours per day. If the profit on each cup A is 75 paise and that on each cup B is 50 paise, show that 15 tea - cups of type A and 30 tea - cups of type B should be manufactured in a day to get the maximum profit. **[CBSE 2003, 2008]**
30. A factory owner purchases two types of machines, A and B for his factory. The

requirements and limitations for the machines are as follows:

	Area occupied by the machine	Labour force for each machine	Daily output in units
Machine A	1000 sq.m	12 men	60
Machine B	1200 sq.m	8 men	40

He has an area of 7600 sq.m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output?

[CBSE 2003, 2008]

31. A gardener has supply of fertilizer of type I which consists of 10% nitrogen and 6% phosphoric acid and type II fertilizer which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type I fertilizer costs 60 paise per kg and type II fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

[CBSE 2002, 2008]

32. A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods x and y . To produce one unit of x , 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of y . If x and y are priced at ₹100 and ₹120 per unit respectively, how should be producer uses his resources to maximize the total revenue? Solve the problem graphically.

[CBSE 2000]

33. A firm manufactures two types of products A and B and sells them at a profit of ₹5 per unit of type A and ₹3 per unit of type B. each product is processed on two machines M_1 and M_2 . One unit of type A requires one

minute of processing time on M_1 and two minutes processing time on M_2 , whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2 . Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

34. A small firm manufactures items A and B. The total number of items A and B that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be ₹300 and one unit item B be ₹160, how many of each type item be produced to maximize the profit? Solve the problem graphically.

35. A company manufactures two types of toys A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours for assembling in a day. The profit is ₹50 each for type A and ₹60 each for type B. How many toys of each type should the company manufacture in a day to maximise the profit?

[CBSE 2001]

36. A company manufactures two articles A and B. there are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of first department is 60 hours a week and that of other department is 48 hours per week. The product of each unit of article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of article B requires 2 hours in assembly and 4 hours in finishing. If the profit is ₹6 for each unit of A and ₹8 for each unit of B, find the number of units of A

and B to be produced per week in order to have maximum profit.

[CBSE 2003]

37. A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and half an hour to make an item B. The maximum time available per day is 16 hours. The profit on an item of A is ₹300 and on one item of B is ₹160. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

[CBSE 2004]

38. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes one hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

[CBSE 2010]

39. A factory makes tennis rackets and cricket bats. A tennis rackets takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time. If the profit on a racket and on a bat is ₹20 and ₹10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve it graphically.

[CBSE 2011]

40. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost ₹25,000 and ₹40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of

units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹70 lakhs and his profit on the desktop model is ₹4500 and on the portable model is ₹5000. Make an LPP and solve it graphically.

[CBSE 2011]

41. A fruit grower can use two type of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

	Kg per bag	
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

42. An oil company requires 12,000, 20,000 and 15,000 barrels of high – grade, medium grade and low grade oil, respectively. Refinery A produces 100, 300 and 200 barrels per day of high – grade, medium – grade and low – grade oil, respectively while refinery B produces 200, 400 and 100 barrels per day of high – grade, medium – grade and low – grade oil, respectively. If refinery A costs ₹400 per day and refinery B costs ₹300 per day to operate, how many days should each run to minimize costs while satisfying requirements.
43. A company produces soft drink that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B to go into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers.

Supplier S has a mix of 4 units of A and 2 units of B that costs ₹10, the supplier T has a mix of 1 unit of A and 1 unit of B that costs ₹4. How many mixes from S and T should the company purchase to honour contract requirement and yet minimize cost? **[CBSE 2012]**

44. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760.00 to invest and has space for at most 20 items. A fan costs him ₹360.00 and a sewing machine ₹240.00. His expectation is that he can sell a fan at a profit of ₹22.00 and a sewing machine at a profit of ₹18.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Translate this mathematically and then solve it.
45. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5,760 to invest and has space for at most 20 items for storages. An electronic sewing machine cost him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated sewing machine at a profit of ₹18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically. **[CBSE 2014]**
46. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packages per day. The postman believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹225 a day and a woman receives ₹200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LLP and solve it graphically. **[CBSE 2015]**
47. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/ kg of vitamin A and 1 unit/ kg of vitamin C. Food 'II' contains 1 unit/ kg of vitamin A and 2 units/ kg of vitamin C. It costs ₹50 per kg to purchase food 'I' and ₹70 per kg to purchase food 'II'. Formulate this problem as a LPP to minimise the cost of such a mixture.
48. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹60 per kg and food Q costs ₹80 per kg. Food P contains 3 units/ kg of vitamin a and 5 units/ kg of vitamin B while food Q contains 4 units/ kg of vitamin A and 2 units/ kg of vitamin B. Determine the minimum cost of the mixture.
49. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of other ingredients used in making the cakes.
50. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹4 per unit food and F_2 costs ₹6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a LPP. Find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements. **[CBSE 2009]**
51. A dietician has to develop a special diet using two foods P and Q. Each packet

(containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A? **[CBSE 2015]**

52. A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹250 per bag, contains 3 units of nutritional element A, 2.5 units of nutritional element B and 2 units of element C. Brand Q costing ₹200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?
53. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs ₹16 and one kg of food Y costs ₹20. Find the least cost of the mixture which will produce the required diet?

54. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/ kg of vitamin A and 1 unit/ kg of

vitamin C. Food 'II' contains 1 unit/ kg of vitamin A and 2 units/ kg of vitamin C. It costs ₹5.00 per kg to purchase food 'I' and ₹7.00 per kg to purchase food 'II'. Formulate the above LPP to minimise the cost of such a mixture and solve it.

[CBSE 2011, 2012]

55. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 of calories. Two foods A and B are available at a cost of ₹4 and ₹3 per unit respectively. If one unit of A contains 200 units of vitamin, 1 unit of mineral and 40 calories and one unit food B contains 100 units of vitamin, 2 units of mineral and 40 calories, find what combination of foods should be used to have the least cost?
56. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of proteins and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs ₹5 per kg and rice costs ₹4 per kg.
57. One kind of cake requires 300g of flour and 15g of fat, and another kind of cake requires 150g of flour and 30g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600gm of fat, assuming that there is no shortage of other ingredients used in making the cakes. Make it as an LPP and solve it graphically. **[CBSE 2010]**
58. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and

Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/ to	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost?

59. An aeroplane can carry a maximum of 200 passengers. A profit of ₹1000 is made on each executive class ticket and a profit of ₹600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?
60. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in ₹)		
From/ To	A	B
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

61. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L

respectively. The distances (in km) between the depots and the petrol pumps is given below in the table:

From/ To	Distance (in km)	
	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

62. If a young man drives his vehicle at 25km/hr, he has to spend ₹2 per km on petrol. If he drives it at a faster speed of 40km/hr, the petrol cost increases to ₹5 per km. He has ₹100 to spend on petrol and travel within one hour. Express this as an LPP and solve the same. [CBSE 2007]
63. A library has to accommodate two different types of books on shelf. The books are 6 cm and 4 cm thick and weigh 1kg and $1\frac{1}{2}$ kg each respectively. The shelf is 96 cm long and at most can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically. [CBSE 2010]