3. MATRICES

1. If a matrix has 18 elements, what are the possible orders it can have? What if it has 5 elements?
2. If a matrix has 24 elements, what are the possible orders it can have? What if it has 13 elements?
3. Construct a 3 × 4 matrix, $A=\left[a_{ij}\right]$, whose elements are given by

   (i) $a_{ij}=\frac{1}{2}\left|-3i+j\right|$ (ii) $a_{ij}=2i-j$
4. Construct a 2 × 2 matrix, $A=\left[a_{ij}\right]$, whose elements are given by:

   (i) $a_{ij}=\frac{(i-j)^2}{2}$ (ii) $a_{ij}=\frac{i-j}{i+j}$
5. If $A=\left[a_{ij}\right]$, where $a_{ij}\begin{cases} i+j, & \text{if } i\ge j \\ i-j, & \text{if } i<j \end{cases}$,

   Construct a 3 × 2 matrix A.
6. Write the element a_{12} of the matrix $A=\left[a_{ij}\right]_{2\times 2}$, whose elements a_{ij} are given by $a_{ij}=e^{2ix}\sin jx$. **[2015]**
7. If $A=\left[a_{ij}\right]=\begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and

   $B=\left[b_{ij}\right]=\begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, then find $a_{22}+b_{21}$.
8. Consider the following information regarding number of notebooks and pens possessed by sudha and her two friends syeeda and simran.

	No. of notebooks	No. of pens
Sudha	15	6
Syeeda	10	2
Simran	13	5

   Represent the above information in the form of a 3 × 2 matrix. What does the entry in the second row and first column represent?
9. The sales figure two car dealers during January 2007 shows that dealer A sold 5 deluxe, 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the two month period of January – February revealed that dealer A sold 8 deluxe, 7 premium and 6 standard cars. In the same two month period, dealer B sold 10 deluxe, 5 premium and 7 standard cars. Write 2 × 3 matrices summarizing sales data for January and 2 month period for each dealer.
10. Find the values of a, b, c and d from the equation:

    $$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$
11. Solve for x and y: $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 11 \end{bmatrix}$
12. Find x, y, z, w, if

    $$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}=3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$
13. Find the value of x, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix}=\begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.
14. Write the values of x – y + z from the following equation:

    $$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$
15. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$. Write the value of x.
16. Simplify:

    $$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}+\sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$
17. Find x, y, a and b, if

    $$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix}=\begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$
18. For what value of x and y are the following matrices equal?

    $$A=\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix}=\begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

19. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c-2 \\ 2b+4 & -21 & 0 \end{bmatrix}$.

Find a, b, c, x, y and z.

20. If $R(\theta)=\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$. Write $R\left(\frac{\pi}{2}\right)$.

21. If $\begin{bmatrix} 2x+1 & 2x \\ 0 & y^2+1 \end{bmatrix}=\begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$. Find the value of $(x+y)$.

22. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix}+\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x-y)$ **[2014]**

23. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}=\begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of $a-2b$. **[2014]**

24. Let $A=\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$. Compute 2A – 3B + 4C.

25. If $A=\text{diag.}\begin{bmatrix} 2 & -5 & 9 \end{bmatrix}$, $B=\text{diag.}\begin{bmatrix} 1 & 1 & -4 \end{bmatrix}$ and $C=\text{diag.}\begin{bmatrix} -6 & 3 & 4 \end{bmatrix}$. Find 2A + 3B – 5C.

26. Find X, if $X+\begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$.

27. Find X, such that A + 2B + X = 0, where $A=\begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}, B=\begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$.

28. Find X and Y if $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

29. Solve the matrix equation:

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix}-3\begin{bmatrix} x \\ 2y \end{bmatrix}=\begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

30. Find X and Y, if $2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y=\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$.

31. If $A=\begin{bmatrix} 1 & 3 & 3 \\ -1 & 0 & 2 \\ 1 & -3 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ and $C=\begin{bmatrix} -1 & -2 & 1 \\ -1 & 2 & 3 \\ -1 & -2 & 2 \end{bmatrix}$. Verify that A + (B + C) = (A + B) + C.

32. Fatima has two factories at places P and Q. each factory produces sport shoes for boys and girls in three different price categories labelled 1,2 and 3. The quantities produced by each factory are represented as matrices given below:

Factory P Factory Q

$$A=\begin{array}{cc} \text{Boys} & \text{Girls} \\ \left[\begin{matrix} 95 \\ 75 \\ 90 \end{matrix}\right. & \left.\begin{matrix} 60 \\ 65 \\ 85 \end{matrix}\right] \end{array}\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \qquad B=\begin{array}{cc} \text{Boys} & \text{Girls} \\ \left[\begin{matrix} 90 \\ 70 \\ 75 \end{matrix}\right. & \left.\begin{matrix} 50 \\ 55 \\ 75 \end{matrix}\right] \end{array}\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Find:

(i) The total production of sports shoes for boys and girls in each price category.

(ii) The difference of sports shoes produced by factories at P and Q for boys and girls in each price category.

(iii) Number of sports shoes produced by factory at P for boys and girls in each price category if production of factory at P (a) reduces to 20% (b) has doubled.

33. Two farmers Ramkishan and Gurcharan singh cultivates only three varieties of rice namely Basmati, Permal, Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September Sales (in Rupees)

$$A=\begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \\ \left[\begin{matrix} 10,000 \\ 50,000 \end{matrix}\right. & \begin{matrix} 20,000 \\ 30,000 \end{matrix} & \left.\begin{matrix} 30,000 \\ 10,000 \end{matrix}\right] \end{array}\begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix}$$

October Sales (in Rupees)

$$B=\begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \\ \left[\begin{matrix} 5,000 \\ 20,000 \end{matrix}\right. & \begin{matrix} 10,000 \\ 10,000 \end{matrix} & \left.\begin{matrix} 6,000 \\ 10,000 \end{matrix}\right] \end{array}\begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix}$$

(i) Find the combined sales in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

34. Two schools P and Q want to award their selected students on the values of Discipline, Politeness and Punctuality. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students with a total award money of ₹ 1,000. School Q wants to spend ₹ 1,500 to award its 4,1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 600, using matrices, find the award money for each value.

Apart from the above three values. Suggest one more value for awards. **[2014]**

35. If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$, then find AB, BA. Prove that AB $\neq$ BA.

36. If $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 0 & 3\\ 3 & -1 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ 0 & 2\\ -1 & 4\end{bmatrix}$ and

$C=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 0 & -2 & 1\end{bmatrix}$

Prove that (AB)C = A(BC).

37. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, Prove that F(x) F(y) = F(x+y).

38. If $\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\ -2 & 4\end{bmatrix}=\begin{bmatrix}-4 & 6\\ -9 & x\end{bmatrix}$, Find x.

39. If $\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$ = 0, Find x.

40. Solve the following matrix equation for x:

$$\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix}=0.$$ **[2014]**

41. Solve for x and y: $\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 11\end{bmatrix}$.

42. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then prove that

$A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$.

43. Find the matrix X, for which

$\begin{bmatrix}5 & 4\\ 1 & 1\end{bmatrix}X=\begin{bmatrix}1 & -2\\ 1 & 3\end{bmatrix}$.

44. Find the matrix X such that,

$\begin{bmatrix}2 & -1\\ 0 & 1\\ -2 & 4\end{bmatrix}X=\begin{bmatrix}-1 & -8 & -10\\ 3 & 4 & 0\\ 10 & 20 & 10\end{bmatrix}$.

45. Three schools, X, Y and Z organized a fete (mela) for collecting funds for floods victims in which they sold hand – held fan, mats and toys made from recycled material, the sale price of each being ₹ 25, ₹ 100 and ₹ 50 respectively. The following table shows the number of articles of each type sold:

School/Article	School X	School Y	School Z
Hand-held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also write any one value generated by the above situation. **[2015]**

46. To raise money for an orphanage, students of three schools A, B and C organized on exhibition in their locality, where they sold paper bags, scrap books and pastel sheets made by them using recycled paper, at the rate of ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap-books and 34 pastel sheets. School B sold 22 paper bags 15 scrap – books and 28 pastel sheets while school sold 26 paper bags, 18 scrap – books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

By such exhibition, which values are generated in the students? **[2015]**

47. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house cells, and letters. The cost per contact (in paise) is given in matrix A as

$$A=\begin{array}{cl} \text{cos t per contact} & \\ \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} & \begin{array}{l} \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{array} \end{array}$$

The number of contacts of each type made in two cities X and Y is given by

$$B=\begin{array}{ccc} \text{Telephone} & \text{House call} & \text{Letter} \\ \end{array}$$
$$B=\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10,000 \end{bmatrix}\begin{array}{l} X \\ Y \end{array}$$

Find the total amount spent by the group in the two cities X and Y.

48. There are families A, B and C. The number of men, women and children in these families are as under.

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are ` 200, ` 150 and ` 200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expanses of each family. What impact does more children in the family create on the society? **[2015]**

49. If $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2-4A-5I=0$ and hence find A^{-1} **[2015]**

50. If $A=\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2-5A+16I$. **[2015]**

51. If $A=\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find a matrix B such that AB = I.

52. If $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x)=x^2-2x-3$, prove that f(A) = 0.

53. If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2-5A+7I=0$. Use this to find A^4.

54. If $A=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then
   (i) Find λ, μ so that $A^2=\lambda A+\mu I$.
   (ii) Prove that $A^3-4A^2+A=0$.

55. If $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that A is a root of the polynomial $f(x)=x^3-6x^2+7x+2$.

56. If $A=\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$. Find A^{16}.

57. If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI+bA)^n=a^nI+na^{n-1}bA$, where n is an integer.

58. Let $A=\begin{bmatrix} 0 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Prove that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

59. If $A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x and y such that $(xI+yA)^2=A$.

60. Prove that the product of matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and

$\begin{bmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{bmatrix}$ is the null matrix, when θ and φ differ by an odd multiple of $\frac{\pi}{2}$.

61. If A is a square matrix such that $A^2 = A$, then write the value of $(I+A)^2 - 3A$.

62. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = 0.

63. If $X_{m\times 3}\, Y_{p\times 3} = Z_{2\times b}$, for true matrices X, Y ,Z. Find the values of m, p, b.

64. If n = p, then find the order of the matrix $A = 7X_{(2\times n)} - 5Z_{(2\times p)}$.

65. A trust fund has `30, 000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% per year. Using matrix multiplication determine how to divide `30, 000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) `1800 (b) `2, 000

66. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then show that $A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$, where n is a positive integer.

67. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of mathematical induction that $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ for n ϵ N.

68. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is a positive integer.

69. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Use the principle of mathematical induction to show that $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ for n ϵ Z.

70. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ where n ϵ N.

71. If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, verify that

(a) $(A+B)' = A' + B'$

(b) $(A')' = A$.

72. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then find the values of θ satisfying the equation $A' + A = I$.

73. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying $AA' = 9I$, then find the values of a and b.

74. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

75. If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$.

76. If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then find A^{-1} using elementary row operations. **[2015]**

77. If $A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, find $A'-B'$.

78. Write a 3×3 skew symmetric matrix.

[2015]

79. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that

(a) $(A+A')$ is a symmetric matrix.

(b) $(A-A')$ is a skew symmetric matrix.

80. Prove that any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix.

81. Let A and B be symmetric matrices of the same order. Then, show that

(a) A + B is a symmetric matrix.

(b) AB – BA is a skew – symmetric matrix.

(c) AB + BA is a symmetric matrix.

82. Express the matrix $A=\begin{bmatrix}3 & 2 & 3\\ 4 & 5 & 3\\ 2 & 4 & 5\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

83. Express the following matrix as the sum of a symmetric and skew – symmetric matrix and verify your result: $\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$.

84. Use elementary column operations $c_2 \to c_2 - 2c_1$ in the matrix equation

$$\begin{bmatrix}4 & 2\\ 3 & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}.$$

[2014]