1. RELATIONS AND FUNCTIONS

RELATIONS

1. Determine whether each of the following relations are reflexive, symmetric and transitive: **[CBSE 2010]**
   - (i) Relation R in the set $A=\{1,2,3,........,13,14\}$ defined as $R=\{(x,y):3x-y=0\}$
   - (ii) Relation R in the set N of natural numbers defined as $R=\{(x,y):y=x+5\, and\; x<4\}$
   - (iii) Relation R in the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y):y\, is\, divisible\, by\, x\}$
   - (iv) Relation R in the set Z of all integers defined as $R=\{(x,y):x-y\; is\; an\; integer\}$
   - (v) Relation R in the set R of all real numbers defined as $R=\{(a,b):a\le b^3\}$
2. Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive.
   - (i) $R=\{(x,y):x\; and\; y\; work\; at\; the\; same\; place\}$
   - (ii) $R=\{(x,y):x\; and\; y\; live\; in\; the\; same\; lacality\}$
   - (iii) $R=\{(x,y):x\; is\; wife\; of\; y\}$
   - (iv) $R=\{(x,y):x\; is\; father\; of\; y\}$
3. Show that the relation R in the set R of real numbers, defined as $R=\{(a,b):a\le b^2\}$ is neither reflexive, nor symmetric nor transitive.
4. Show that the relation R in R defined as $R=\{(a,b):a\le b\}$, is reflexive and transitive but not symmetric.
5. Show that the relation R on the set A = {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.
6. Let A {1, 2, 3}. Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is four.
7. Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.
8. Let $R=\{(a,a^3): a$ is a prime number less than $5\}$ be a relation. Find the range of R. **[2014]**
9. Let R be a relation defined on the set of natural numbers N as $R=\{(x,y):x,y\in N, 2x+y=41\}$. Find the domain and range of R. Also, verify whether R is (i) reflexive (ii) symmetric (iii) transitive.
10. Give an example of a relation, which is
    - (i) Symmetric but neither reflexive nor transitive.
    - (ii) Transitive but neither reflexive nor symmetric.
    - (iii) Reflexive and symmetric but not transitive.
    - (iv) Reflexive and transitive but not symmetric.
    - (v) Symmetric and transitive but not reflexive.
11. Show that each of the relation R in the set $A=\{x\in Z:0\le x\le 12\}$, given by
    - (i) $R=\{(a,b):|a-b|\; is\; a\; multiple\; of\; 4\}$ **[CBSE 2010]**

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(ii) $R=\{(a,b):a=b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

12. Show that the relation R on the set A = {1, 2, 3, 4, 5}, given by $R=\{(a,b):|a-b|\ is\ even\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. **[CBSE 2009, 2015]**
13. Prove that the relation R on Z defined by $(a,b)\in R \Leftrightarrow a-b\ is\ divisible\ by\ 5$ is an equivalence relation on Z. **[CBSE 2010]**
14. Show that the relation R on the set A of points in a plane, given by R = {(P, Q) : Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.
15. Prove that the relation R on the set $N\times N$ defined by $(a,b)R(c,d)\Leftrightarrow a+d=b+c$ for all $(a,b),(c,d)\in N\times N$ is an equivalence relation. **[CBSE 2010]**
16. Let $A=\{1,2,3,\ldots..,9\}$ and R be the relation in $A\times A$ defined by $(a,b)R(c,d)$ if $a+d=b+c$ for $(a,b),(c,d)$ in $A\times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$. **[2014]**
17. Let N be the set of all natural numbers and let R be a relation on $N\times N$, defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ for all $(a,b),(c,d)\in N\times N$. Show that R is an equivalence relation on $N\times N$.
18. Show that the relation R defined in the set A of all triangles as $R=\{(T_1,T_2):T_1\ is\ similar\ to\ T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related ?
19. Show that the relation R defined in the set A of all polygons as $R=\{(P_1,P_2):P_1\ and\ P_2\ have\ same\ number\ of\ sides\}$, is an equivalence relation. What is the set of all elements in A related to the right angled triangle T with sides 3, 4 and 5 ?
20. Let L be the set of all lines in XY plane and R be the relation in L defined as $R=\{(L_1,L_2):L_1\ is\ parallel\ to\ L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y=2x+4$.
21. Show that the relation R defined by $R=\{(a,b):a-b\ is\ divisible\ by\ 3; a,b\in Z\}$ is an equivalence relation. **[CBSE 2008]**
22. Show that the relation R on Z defined by $R=\{(a,b):2\ divides\ a-b\}$, is an equivalence relation.

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FUNCTIONS

1. Let A be the set of all 50 students of class XII in a Central School. Let $f : A \to N$ be a function defined by f(x) = Roll number of student x. Show that f is one-one but not onto.
2. Show that the function $f : N \to N$, given by $f(x) = 2x$, is one-one but not onto.
3. Check the injectivity and surjectivity of the following functions :
   (i) $f : N \to N$ given by $f(x) = x^2$
   (ii) $f : Z \to Z$ given by $f(x) = x^2$
   (iii) $f : R \to R$ given by $f(x) = x^2$
   (iv) $f : Z \to Z$ given by $f(x) = x^3$
   (v) $f : R \to R$ given by $f(x) = |x|$
   (vi) $f : Z \to Z$ given by $f(x) = x - 5$
   (vii) $f : R \to R$ given by $f(x) = \sin x$
   (viii) $f : Z \to Z$ given by $f(x) = x^2 + x$
   (ix) $f : R \to R$ given by $f(x) = \sin^2 x + \cos^2 x$
   (x) $f : Q - \{3\} \to Q$ given by $f(x) = \frac{2x+3}{x-3}$
4. Prove that the Greatest Integer Function $f : R \to R$, given by $f(x) = [x]$, is neither one-one nor onto.
5. Show that the function $f : R - \{3\} \to R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection. **[CBSE 2012]**
6. Show that the function $f : R \to R$ given by $f(x) = ax + b,\ where\ a, b \in R, a \neq 0$ is a bijection. **[CBSE 2010]**
7. Let A = R – {2} and B = R – {1}. If $f : A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is a bijective.
8. Let A and B be sets. Show that $f : A \times B \to B \times A$ such that $f(a,b) = (b,a)$ is bijective function.
9. Show that $f : N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$ is onto but not one-one. **[CBSE 2009]**
10. If $f : R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection. **[CBSE 2011]**
11. Let $f : N \to N$ be defined by $f(n) = \begin{cases} n+1, if\ n\ is\ odd \\ n-1, if\ n\ is\ even \end{cases}$. Show that f is a bijection. **[CBSE 2012]**
12. Let R be the set of real numbers. If $f : R \to R$; $f(x) = x^2$ and $g : R \to R$; $g(x) = 2x + 1$. Then, find $fog\ and\ gof$. Also, show that $fog \neq gof$.

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13. Find fog and gof when $f: R \to R$ and $g: R \to R$ are defined by
   (i) $f(x) = 2x+3$ and $g(x) = x^2 + 5$
   (ii) $f(x)$ and $g(x) = |x|$
   (iii) $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$
   (iv) $f(x) = \sin x$ and $g(x) = x^2$
14. Let $f: \{2,3,4,5\} \to \{3,4,5,9\}$ and $g: \{3,4,5,9\} \to \{7,11,15\}$ be functions defined as $f(2)=3,\ f(3)=4,\ f(4)=f(5)=5$ and $g(3)=g(4)=7$ and $g(5)=g(9)=11$. Find gof.
15. Let $f: \{1,3,4\} \to \{1,2,5\}$ and $g: \{1,2,5\} \to \{1,3\}$ be given by $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(1,3),(2,3),(5,1)\}$. Write down gof.
16. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.
17. If $f: R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Show that $gof = I_A$ and $fog = I_B$, where $B = R - \left\{\frac{3}{5}\right\}$ and $A = R - \left\{\frac{7}{5}\right\}$.
18. Let f, g and h be functions from R to R. Show that $\begin{matrix}(f+g)oh = foh + goh \\ (f.g)oh = (foh).(goh)\end{matrix}$.
19. Consider $f: N \to N, g: N \to N$ and $h: N \to R$ defined as $f(x) = 2x,\ g(y) = 3y+4,\ h(z) = \sin z$ for all $x, y, z \in N$. Show that $ho(gof) = (hog)of$.
20. Let $f: Z \to Z$ be defined by f(n) = 3n for all $n \in Z$ and $g: Z \to Z$ be defined by

$$g(n) = \begin{cases} \frac{n}{3}, & \text{if } n \text{ is a multiple of } 3 \\ 0, & \text{if } n \text{ is not a multiple of } 3 \end{cases} \quad \text{for all } n \in Z.$$

   Show that $gof = I_Z$ and $fog \neq I_Z$
21. Give examples of two functions $f: N \to N$ and $g: N \to N$ such that gof is onto but f is not onto.
22. Give examples of two functions $f: N \to Z$ and $g: Z \to Z$ such that gof is injective but g is not injective.
23. If $f: Q \to Q$ is given by $f(x) = x^2$, then find
   (i) $f^{-1}(9)$ (ii) $f^{-1}(-5)$ (iii) $f^{-1}(0)$.
24. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

[CBSE 2012, 13]

25. Show that $f:[-1,1]\to R$ given by $f(x)=\frac{x}{x+2}$ is one-one. Find the inverse of the function $f:[-1,1]\to Range\ f$.

26. Consider $f:R\to R$ given by $f(x)=4x+3$. Show that *f* is invertible. Find the inverse of *f*. **[2015]**

27. Let $f:W\to W$, be defined as $f(x)=x-1$, if x is odd and $f(x)=x+1$, if x is even. Show that f is invertible. Find the inverse of f, where W is the set of all whole numbers. **[2014]**

28. Let $f:N\to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f:N\to Range(f)$ is invertible. Find the inverse of *f*. **[CBSE 2010]**

29. Let $f:R\to R$ be defined as $f(x)=10x+7$. Find the function $g:R\to R$ such that $gof=fog=I_R$. **[CBSE 2011]**

30. Consider $f:R^+\to[4,\infty)$ given by $f(x)=x^2+4$. Show that *f* is invertible with inverse f^{-1} of *f* given by $f^{-1}(x)=\sqrt{x-4}$, where R+ is the set of all non-negative real numbers. **[CBSE 2013]**

31. Consider $f:R^+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(x)=\frac{\sqrt{x+6}-1}{3}$.

32. Let A = R – {3} and B = R – {1}. Consider the function $f:A\to B$ defined by $f(x)=\frac{x-2}{x-3}$. Show that *f* is one-one and onto and hence find f^{-1}. **[CBSE 2012]**

33. Consider $f:\{1,2,3\}\to\{a,b,c\}$ and $g:\{a,b,c\}\to\{apple, ball, cat\}$ defined as $f(1)=a,\ f(2)=b,\ f(3)=c,\ g(a)=apple,\ g(b)=ball,\ g(c)=cat$. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(gof)^{-1}$ and show that $(gof)^{-1}=f^{-1}og^{-1}$

34. If $f:R\to R$ be given by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then find $fof(x)$.

35. If the function $f:R\to R$ be defined by $f(x)=2x-3$ and $g:R\to R$ by $g(x)=x^3+5$, find the value of $(fog)^{-1}(x)$. **[2015]**

36. Let $f:R\to R$ be the signum function defined as $f(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0\end{cases}$ and $g:R\to R$ be the Greatest Integer Function given by $g(x)=[x]$. Then does fog and gof coincide in $(0,1]$?

37. Show that the function $f:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}, x\in R$ is one-one and onto function.

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Binary Operations

1. Determine whether or not each of the definition of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this.
   (i) $On\ Z^+, define * by\ a*b = a-b$
   (ii) $On\ Z^+, define * by\ a*b = ab$
   (iii) $On\ R, define * by\ a*b = ab^2$
   (iv) $On\ Z^+, define * by\ a*b = |a-b|$
   (v) $On\ Z^+, define * by\ a*b = a$
   (vi) $On\ N, define * by\ a*b = a^b$
   (vii) $On\ N, define * by\ a*b = a+b-2$
   (viii) $On\ Q, define * by\ a*b = \frac{a-1}{b+1}$
2. Let $*$ be a binary operation on the set I of integers, defined by $a*b = 2a+b-3$. Find the value of $3*4$. **[CBSE 2011]**
3. Is $*$ defined on the set {1, 2, 3, 4, 5} by $a*b = LCM\ of\ a\ and\ b$ a binary operation ? Justify your answer.
4. The binary operation $*: R\times R \to R$ is defined as $a*b = 2a+b$. Find $(2*3)*4$. **[CBSE 2012]**
5. Let $*$ be a binary operation, on the set of all non-zero real numbers, given by $a*b = \frac{ab}{5}$ for all $a, b \in R-\{0\}$. Find the value of x, given that $2*(x*5) = 10$. **[2014]**
6. Let $*$ be a binary operation on N given by $a*b = LCM(a,b)\ for\ all\ a,b \in N$. Find $5*7$. **[CBSE 2012]**
7. Consider the binary operation $\wedge$ on the set {1, 2, 3, 4, 5} defined by $a \wedge b = \min\{a,b\}$. Write the operation table of the operation $\wedge$.
8. Check the commutativity and associativity of each of the following binary operations :
   (i) $*$ on Z defined by $a*b = a+b+ab$
   (ii) $*$ on Q defined by $a*b = a^2+b^2$
   (iii) $*$ on Q defined by $a*b = a+ab$
   (iv) $*$ on Q defined by $a*b = \frac{ab}{4}$
   (v) $*$ on R defined by $a*b = a+b-7$
   (vi) $*$ on Q defined by $a*b = ab+1$
   (vii) $*$ on N defined by $a*b = a^b$
9. Determine which of the following binary operations are associative and which are commutative :
   (i) $*$ on N defined by $a*b = 1\ for\ all\ a,b \in N$
   (ii) $*$ on Q defined by $a*b = \frac{a+b}{2}\ for\ all\ a,b \in Q$ **[CBSE 2008]**
10. Let $*$ be the binary operation on N given by $a*b = LCM\ of\ a\ and\ b$. Find

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(i) $5*7,\ 20*16$

(ii) Is $*$ commutative ?

(iii) Is $*$ associative ?

(iv) Find the identity of $*$ in N.

11. Let $*$ be the binary operation on N defined by $a*b = HCF\ of\ a\ and\ b$. Is $*$ commutative? Is associative? Does there exist identity for this binary operation on N?

12. Consider the binary operations $*: R\times R \to R\ and\ o: R\times R \to R$ defined as $a*b = |a-b|$ and $aob = a$ for all $a, b\in R$. Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that $*$ is distributive over o. Does o distributive over $*$? Justify your answer. **[CBSE 2012]**

13. Let $A = N\times N$ and $*$ be the binary operation on A defined by $(a,b)*(c,d) = (a+c, b+d)$ Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

6. Let $A = Q\times Q$, where Q is the set of all relations numbers, and $*$ binary operation defined on A by $(a,b)*(c,d) = (ac, b+ad)$, for all $(a,b),(c,d)\in A$.

Find

(i) The identity element in A.

(ii) The invertible element of A. **[2015]**

14. If $*$ is defined on the set R of real numbers by $a*b = \frac{3ab}{7}$, find the identity element in R for the binary operation $*$. **[CBSE 2012]**

15. If the binary operation $*$ on the set Z is defined by $a*b = a+b-5$, then find the identity element with respect to $*$. **[CBSE 2012]**

16. Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as $a*b = \begin{cases} a+b, & if\ a+b<6 \\ a+b-6, & if\ a+b\geq 6 \end{cases}$.

Show that zero is the identity for this operation and each element a of the set is invertible with 6 – a being the inverse of a.

17. Let X be a non-empty set and let $*$ be a binary operation on P(X) defined by $a*b = A\cup B$ for all $A, B\in P(X)$. Prove that $*$ is both commutative and associative on P(X). Find the identity element with respect to $*$ on P(X). Show that $\varphi\in P(X)$ is the only invertible element of P(X).

18. Let X be a non-empty set and let $*$ be a binary operation on P(X) defined by $a*b = A\cap B$ for all $A, B\in P(X)$.

(i) Find the identity element with respect to $*$ in P(X).

(ii) Show that X is the only invertible element of P(X).

19. Let $A = Q\times Q$, where Q is the set of all rational numbers, and $*$ be a binary operation on A defined by $(a,b)*(c,d) = (ac, b+ad)$ for $(a,b),(c,d)\in A$. Then find **[2015]**

(i) The identify elements of $*$ in A.

(ii) Invertible elements of A, and hence write the inverse of elements $(5,3)$ and $\left(\frac{1}{2}, 4\right)$.

20. Let $A = N\cup\{0\}\times N\cup\{0\}$ and let $*$ be a binary operation on A defined by $(a,b)*(c,d) = (a+c, b+d)\ for\ all\ (a,b),(c,d)\in A$. Show that :

(i) $*$ is commutative on A

(ii) $*$ is associative on A

Also, find the identity element, if any, in A.

21. Let $A = N \times N$, and let $*$ be a binary operation on A defined by $(a,b)*(c,d) = (ad+bc, bd)$ $for\ all$ $(a,b),(c,d) \in N \times N$. Show that
    (i) $*$ is commutative on A
    (ii) $*$ is associative on A
    (iii) A has no identity element.
22. Show that the number of binary operations on {1, 2} having 1 as identity and having 2 as inverse of 2 is exactly one.

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