# 10. VECTOR ALGEBRA

1. Represent graphically
   (a) A displacement of 40 km, 30° west of south.
   (b) 60 km, 40° east of north.
   (c) 50 km, south – east.
2. Classify the following measures as scalars and vectors
   (a) 10 kg (b) 10 metres north – west
   (c) 30 km/hr (d) 50 m/sec towards north
   (e) Newton (f) $10^{-19}$ coloumb
3. Represent the following graphically:
   (a) A displacement of 40 km, 30° east of north.
   (b) A displacement of 50 km south – east.
   (c) A displacement of 70 km, 40° north of west.
4. Classify the following measures as scalars and vectors.
   (a) 15 kg (b) 20 kg weight
   (c) 45° (d) 50 m/sec²
   (e) 10 metres south – east.
5. Classify the following as scalars and vector quantities:
   (a) Time period (b) distance
   (c) Displacement (d) Force
   (e) Work (f) Velocity
   (g) Acceleration
6. Answer the following as true or false:
   (a) $\vec{a}$ and $\vec{a}$ are collinear.
   (b) Two collinear vectors are always equal in magnitude.
   (c) Zero vector is unique.
   (d) Two vectors having same magnitude are collinear.
   (e) Two collinear vectors having the same magnitude are equal.
7. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be the vectors represented by the sides of a triangle, taken in order, then prove that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$.
8. Given a condition that three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the three sides of a triangle. What are other possibilities?
9. If $P_1$, $P_2$, $P_3$, $P_4$ are points in a plane or space and O is the origin of vectors, show that $P_4$ coincides with O iff $\overrightarrow{OP_1}+\overrightarrow{P_1P_2}+\overrightarrow{P_2P_3}+\overrightarrow{P_3P_4}=\vec{0}$.
10. If P, Q, R are three collinear points such that $\overrightarrow{PQ}=\vec{a}$ and $\overrightarrow{QR}=\vec{b}$. Find the vector $\overrightarrow{PR}$
11. If $\overrightarrow{PO}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$, show that the points P, Q, R are collinear.
12. If $\vec{a},\vec{b}$ are any two vectors, then give the geometrical interpretation of the relation $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$.
13. If sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.
14. Find a vector in the direction of vector $2\hat{i}-3\hat{j}+6\hat{k}$ which has magnitude 21 units. **[2014]**
15. If $\vec{a}$ and $\vec{b}$ represent two adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively of a parallelogram ABCD, then show that its diagonals $\overrightarrow{AC}$ and $\overrightarrow{DB}$ are equal to $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ respectively.
16. Vectors drawn from the origin O to the points A, B and C are respectively $\vec{a}$, $\vec{b}$ and $4\vec{a}-3\vec{b}$. Find $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
17. A, B, P, Q and R are five points in a plane. Show that the sum of the vectors $\overrightarrow{AP}$, $\overrightarrow{AQ}$, $\overrightarrow{AR}$, $\overrightarrow{PB}$, $\overrightarrow{QB}$ and $\overrightarrow{RB}$ is $3\overrightarrow{AB}$.
18. For any two vectors $\vec{a}$ and $\vec{b}$, prove that
   (a) $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$
   (b) $|\vec{a}-\vec{b}|\le|\vec{a}|+|\vec{b}|$
   (c) $|\vec{a}-\vec{b}|\ge|\vec{a}|-|\vec{b}|$
19. If $\vec{a}$ and $\vec{b}$ are two non – collinear vectors having the same initial point. What are the vectors represented by $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.



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20. ABCD is a quadrilateral. Find the sum of vectors $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$.
21. If P is a point and ABCD is a quadrilateral and $\overrightarrow{AP}+\overrightarrow{PB}+\overrightarrow{PD}=\overrightarrow{PC}$, show that ABCD is a parallelogram.
22. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct non – zero vectors represented by directed line segments from the origin of the points A, B, C and D respectively, and if $\vec{b}-\vec{a} = \vec{c}-\vec{d}$, then prove that ABCD is a parallelogram.
23. ABCDE is a pentagon, prove that
    (a) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EA}=\vec{0}$.
    (b) $\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}=3\overrightarrow{AC}$.
24. Five forces $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}$ and $\overrightarrow{AF}$ act at the vertex of a regular hexagon ABCDEF. Prove that the resultant is 6 $\overrightarrow{AO}$, where O is the centre of hexagon.
25. If $\vec{a}$ and $\vec{b}$ are the vectors determined by two adjacent sides of a regular hexagon, what are the vectors determined by the other sides taken in order?
26. Let O be the centre of a regular hexagon ABCDEF. Find the sum of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$, $\overrightarrow{OE}$ and $\overrightarrow{OF}$.
27. Prove that the sum of all vectors drawn from the centre of a regular octagon to its vertices is the zero vector.
28. If $\vec{a}$ is a vector and m is a scalar such that $m\vec{a}=\vec{0}$, then what are the alternatives for m and $\vec{a}$.
29. Find the position vectors of the points which divide the join of the points $2\vec{a}-3\vec{b}$ and $3\vec{a}-2\vec{b}$ internally and externally in the ratio 2 : 3.
30. Find the position vector of a point R which divides the line segment joining P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$, externally in the ratio 1 : 2. Also, show that P is the mid – point of the line segment RQ. **[CBSE 2010]**
31. If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then find the position vector of points of trisection of AB.
32. Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are such that $3\vec{a}-\vec{b}+2\vec{c}-4\vec{d}=\vec{0}$. Show that the four points are coplanar. Also, find the position vector of the point of intersection of lines AC and BD.
33. If D is the mid – point of the side BC of a triangle ABC, prove that $\overrightarrow{AB}+\overrightarrow{AC}=2\overrightarrow{AD}$.
34. Prove using vectors: Medians of a triangle are concurrent.
35. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$.
36. If D and E are mid – points of sides AB and AC of a triangle ABC respectively. Show that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{3}{2}\overrightarrow{BC}$.
37. Using vector method, prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.
38. Prove that the sum of the vectors directed from the vertices to the mid points of opposite sides of a triangle is zero.
39. Prove using vectors: The diagonals of a quadrilateral bisect each other if it is a parallelogram.
40. Using vector method, prove that the line segments joining the mid points of the adjacent sides of a quadrilateral taken in order from a parallelogram.
41. Prove by vector method that the line segment joining the mid points of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.
42. ABCD is a parallelogram. E, F are mid – points of BC and CD respectively. AE, AF meet the diagonal BD at points Q and P respectively. Show that points P and Q trisect DB.
43. If P and Q are the mid points of the sides AB and CD of a parallelogram ABCD, prove

that DP and BQ cut the diagonal AC in its points of trisection which are also the points of trisection of DP and BQ respectively.

44. Using vector method, prove that the lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.
45. If O is a point in space, ABC is a triangle and D, E, F are the mid points of the sides BC, CA and AB respectively of the triangle, prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}$.
46. Show that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.
47. ABCD is a parallelogram and P is the point of intersection of its diagonals. If O is the origin of reference, show that
$$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\overrightarrow{OP}.$$
48. ABCD are four points in a plane and Q is the point of intersection of the lines joining the mid – points of AB and CD; BC and AD. Show that $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}=4\overrightarrow{PQ}$, where P is any point.
49. Prove by vector method that the internal bisectors of the angles of a triangle are concurrent.
50. Prove that, for any three vectors $\vec{a}, \vec{b}, \vec{c}$
$$\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a},\ \vec{b},\ \vec{c}\right]$$
**[2014]**
51. Show that the four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively such that $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=\vec{0}$, are coplanar. Also, find the position vector of the point of intersection of the lines AC and BD.
52. Find $x$ such that the four points $A(4,1,2), B(5,x,6), C(5,1,-1)$ and $D(7,4,0)$ are coplanar.
53. Show that the line segments joining the mid – points of opposite sides of a quadrilateral bisects each other.
54. Find the values of x, y and z so that the vectors $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ and $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ are equal.
55. Find the sum of the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$
56. Find the magnitude of the vector $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$.
57. Find the unit vector in the direction of $3\hat{i}-6\hat{j}+2\hat{k}$.
58. Find the unit vector in the direction of $\vec{a}+\vec{b}$, if $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$.
59. Find the unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.
60. Find a unit vector in the direction of the resultant of the vectors $\hat{i}-\hat{j}+3\hat{k}$, $2\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$.
61. Write a unit vector in the direction of $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$. **[CBSE 2008]**
62. Write a unit vector in the direction of $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$. **[CBSE 2009]**
63. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}+9\hat{k}$, find a unit vector parallel to $\vec{a}+\vec{b}$. **[CBSE 2008]**
64. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.
65. Using vector method, find the distance between the points A (1, 2, 3), B (– 1, 2, – 3).
66. The position vectors of the points P, Q, R are
$\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}+5\hat{k}$ and $7\hat{i}-\hat{k}$
respectively. Prove that P, Q and R are collinear.
67. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find

unit vectors parallel to the diagonals of the parallelogram.

68. The adjacent sides of a parallelogram are represented by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}+2\hat{k}$. Find the unit vectors parallel to the diagonals of the parallelogram.
69. If the position vectors of the points A, B, C, D are $2\hat{i}+4\hat{k}$, $5\hat{i}+3\sqrt{3}\hat{j}+4\hat{k}$, $-2\sqrt{3}\hat{j}+\hat{k}$ and $2\hat{i}+\hat{k}$ respectively, prove that CD is parallel to AB and $CD=\frac{2}{3}AB$.
70. Show that the points A, B and C with position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively, form the vertices of a right angled triangle.
71. If $\overline{PQ}=3\hat{i}+2\hat{j}-\hat{k}$ are the coordinates of P are $(1,-1,2)$, find the coordinates of Q.
72. Prove that the points $\hat{i}-\hat{j}$, $4\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+5\hat{k}$ are the vertices of a right – angled triangle.
73. Show that the points $A\left(2\hat{i}-\hat{j}+\hat{k}\right)$, $B\left(\hat{i}-3\hat{j}-5\hat{k}\right)$, $C\left(3\hat{i}-4\hat{j}+4\hat{k}\right)$ are the vertices of a right – angled triangle.
74. Find the vector from the origin O to the centroid of the triangle whose vertices are $(1,-1,2)$, $(2,1,3)$ and $(-1,2,-1)$.
75. Find the position vector of a point R which divides the line segment joining points $P\left(\hat{i}+2\hat{j}+\hat{k}\right)$ and $Q\left(-\hat{i}+\hat{j}+\hat{k}\right)$ in the ratio 2:1.
(a) Internally (b) Externally
76. Find the position vector of a point R which divides the line joining the two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also show that P is the midpoint of the line segment RQ.
77. Find the position vector of the mid – point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).
78. Find the value of x for which $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector.
79. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$.
80. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$ **[2011]**
81. Show that the points $A(1,-2,-8)$, $B(5,0,-2)$ and $C(11,3,7)$ are collinear, and find the ratio in which B divides AC.
82. Find a vector of magnitude of 5 units parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$. **[CBSE 2011]**
83. A vector $\overline{OP}$ is inclined to OX at 45° and OY at 60°. Find the angle at which $\overline{OP}$ is inclined to OZ.
84. If a vector makes angles α, β, γ with OX, OY and OZ respectively, prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$.
85. Find the direction cosines of a vector $\vec{r}$ which is equally inclined with OX, OY and OZ. If $|\vec{r}|$ is given, find the total number of such vectors.
86. A vector $\vec{r}$ has length 21 and direction ratios $2,-3,6$. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ make an acute angle with x – axis.
87. Find the angles at which the vector $2\hat{i}-\hat{j}+2\hat{k}$ is inclined to each of the coordinate axes.
88. Find the direction cosines of the vector joining the points $A(1,2,-3)$ and $B(-1,-2,1)$, directed from A to B.
89. Can a vector have direction cosines 45°, 60°, 120°?

90. Prove that 1, 1, 1 cannot be direction cosines of a straight line.

91. A vector makes an angle $\frac{\pi}{4}$ with each of the x – axis and y – axis. Find the angle made by it with the z – axis.

92. If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $k$, then find θ and hence, the components of $\vec{a}$.

93. A vector $\vec{r}$ is inclined at equal acute angles to x – axis, y – axis and z – axis. If $\left|\vec{r}\right| = 6$ units, find $\vec{r}$.

94. A vector $\vec{r}$ is inclined to x – axis at 45° and y – axis at 60°. If $\left|\vec{r}\right| = 8$ units, find $\vec{r}$.

95. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined with the axes OX, OY and OZ.

96. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

97. If $\vec{a}, \vec{b}, \vec{c}$ represent the sides of a triangle taken in order, then write the value of $\vec{a}+\vec{b}+\vec{c}$ **[CBSE 2011]**

98. Find the position vector of the midpoint of the line segment AB, where A is the point $(3, 4, -2)$ and B is the point $(1, 2, 4)$

99. Find a vector in the direction of $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$, which has a magnitude of 6 units. **[CBSE 2010]**

100. What is the cosine of the angle which the vector $\sqrt{2}\hat{i}+\hat{j}+2\hat{k}$ makes with y – axis? **[CBSE 2010]**

101. For what value of 'a' the vector $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}+6\hat{j}-8\hat{k}$ are collinear? **[CBSE 2011]**

102. Find the sum of the following vectors: $\vec{a}=\hat{i}-2\hat{j}$, $\vec{b}=2\hat{i}-3\hat{j}$, $\vec{c}=2\hat{i}+3\hat{k}$ **[CBSE 2012]**

103. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\vec{a}.\vec{b}=1$.

104. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b}=\sqrt{6}$.

105. Find angle 'θ' between the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

106. Find the angle between the vector $\hat{i}-\hat{j}$ and $\hat{j}-\hat{k}$. **[CBSE 2015]**

107. If $\vec{a}$ and $\vec{b}$ are two unit vector such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. **[CBSE 2014]**

108. Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

109. Vectors $\vec{a}, \vec{b}$ $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{o}$ and $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$ and $\left|\vec{c}\right|=7$. Find the angle between $\vec{a}$ and $\vec{b}$. **[2014]**

110. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

111. Find the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

112. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

113. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

114. Write the projection of vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$. **[CBSE 2014]**

115. Write the projection of vector $2\hat{i}+3\hat{j}-\hat{k}$ along the vector $\hat{i}+\hat{j}$. **[CBSE 2015]**

116. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$.

**[CBSE 2014]**

117. Find $|\vec{a}-\vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$.

118. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

119. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$, then find $|\vec{x}|$.

120. Find $|\vec{x}|$, if for a unit vector $\vec{a}$ $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$.

121. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k})$, $B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-5\hat{k})$ are collinear.

122. Show that each of the given three vectors is a unit vector:
$\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}), \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}), \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$.
Also show that they are mutually perpendicular to each other.

123. Evaluate the product $(3\vec{a}-5\vec{b}).(2\vec{a}+7\vec{b})$.

124. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is $60°$ and their scalar product is $\frac{1}{2}$.

125. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of $\lambda$.

126. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

127. If $\vec{a}.\vec{a}=0$ and $\vec{a}.\vec{b}=0$, then what can be concluded about the vector $\vec{b}$ ?

128. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

129. If the vertices A, B, C of a triangle ABC is $(1,2,3),(-1,0,0),(0,1,2)$ respectively, then find $\angle ABC$.

130. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$

131. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.

132. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. **[CBSE 2014]**

133. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

134. Find the area of a triangle having the points $A(1,1,1), B(1,2,3)$ and $C(2,3,1)$ as its vertices.

135. Find the area of a triangle having the points $A(1,1,2), B(2,3,5)$ and $C(1,5,5)$ as its vertices.

136. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

137. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

138. Write all the unit vectors in XY – plane.

139. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

140. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two vectors, prove that $|\vec{a}+\vec{b}+\vec{c}|=5\sqrt{2}$. Also find $|\vec{a}+\vec{b}+\vec{c}|$.

**[CBSE 2010]**

141. Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|=1$, $|\vec{b}|=4$ and $|\vec{c}|=2$.

142. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha}=3\hat{i}-\hat{j}$, $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. **[CBSE 2012]**

143. Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

144. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$.

145. For any two vectors $\vec{a}$ and $\vec{b}$, prove that: $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2 \Leftrightarrow \vec{a},\vec{b}$ are orthogonal.

146. Show that the projection vector of $\vec{a}$ on $\vec{b}$ $(\neq 0)$ (component of $\vec{a}$ along $\vec{b}$) is $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

147. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal. **[CBSE 2004]**

148. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ **[CBSE 2007]**

149. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$ **[CBSE 2001, 2009]**

150. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$. **[CBSE 2001, 2004]**

151. For any two vectors $\vec{a}$ and $\vec{b}$, show that:
$$\left(1+|\vec{a}|^2\right)\left(1+|\vec{b}|^2\right)=\left|\left(1-|\vec{a}|.|\vec{b}|^2\right)\right|+\left|\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right|^2$$
**[CBSE 2002]**

152. Evaluate $\left[\hat{i}\ \hat{j}\ \hat{k}\right]\left[\hat{i}\ \hat{k}\ \hat{j}\right]$.

153. Find $\lambda$ so that the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar.

154. Show that four points whose position vectors are $6\hat{i}-7\hat{j}$, $16\hat{i}-29\hat{j}-4\hat{k}$, $3\hat{j}-6\hat{k}$, $2\hat{i}+5\hat{j}+10\hat{k}$ are coplanar.

155. If the vectors $\vec{\alpha}=a\hat{i}+\hat{j}+\hat{k}$, $\vec{\beta}=\hat{i}+b\hat{j}+\hat{k}$ and $\vec{\gamma}=\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$, where $a\neq1$, $b\neq1$ and $c\neq1$

156. Find the altitude of a parallelopiped determined by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, if the base is taken to the parallelogram determined by $\vec{a}$ and $\vec{b}$, and if $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{a}=\hat{i}+\hat{j}+3\hat{k}$

157. $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of points A, B, C; prove that $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane of triangle ABC.

158. Show that the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar if and only if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar. **[CBSE 2014]**

159. Write the vector of $\hat{i}\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{k}\times\hat{i}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$ **[2015]**

160. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=5\hat{i}-4\hat{j}+3\hat{k}$, than find the value of $\left(\vec{a}+\vec{b}\right).\vec{c}$