FULL LENGTH TEST

1. All questions are compulsory
2. Question numbers **1** to **6** carry **1 mark** each.
3. Question numbers **7** to **19** carry **4 marks** each
4. Question numbers **20** to **26** carry **6 marks** each.

SECTION – A

1. To position vector of points A and B are $\vec{a}$ and $\vec{b}$ respectively. P divides AB in the ratio 3 : 1 and Q is mid-point of AB. Find the position vector of Q.
2. Find the area of the parallelogram, whose diagonals are $\vec{d_1} = 5\hat{i}$ and $\vec{d_2} = 2\hat{j}$
3. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.
4. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, Write the cofactor of a_{32} (the element of third row and 2nd column).
5. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m+n.
6. Write the differential equation representing the curve $y^2 = 4ax$ where a is an arbitrary constant.

SECTION – B

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at rate of Rs. 20, Rs 15 and Rs. 5 per unit respectively. School A sold 25 paper bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.
   By such exhibition, which values are inculcated in the students?
8. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$ then show that $A^2 - 4A + 7I = O.$
   Using this result calculate A^3 also.

   OR

   If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$ find A^{-1}, using elementary row operations.
9. If x, y, z are in GP, then using properties of determinants, show that
   $\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix} = 0$ where $x \neq y \neq z$ and p is any real number.
10. Evaluate: $\int_{-1}^{1} |x\cos \pi x|\, dx.$
11. Evaluate: $\int \frac{1+\sin 2x}{1+\cos 2x} . e^{2x}\, dx.$

    OR

    Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$
12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'.

    OR

    How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%

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13. For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ if $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}\times\vec{c}=\vec{b}$, then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}|=|\vec{a}|$ and $|\vec{a}|=1$
14. Find the equation of the line through the point (1, 1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, - 1, 0) and (1, 2, - 1), (2, 1, 1)

OR

Find the position vector of the foot of perpendicular drawn from the point P(1, 8, 4) to the line joining A(0, - 1, 3) and B(5, 4, 4). Also find the length of this perpendicular.

15. Solve for $x : \sin^{-1} 6x + \sin 6\sqrt{3} = -\frac{\pi}{2}$

OR

Prove that $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

16. If $x=\sin t$, $y=\sin kt$, show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$
17. If $y^x + x^y + x^x = a^b$, find $\frac{dy}{dx}$
18. It is given that for the function $f(x)=x^3+bx^2+ax+5$ on [1, 3], Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$

    Find values of a and b
19. Evaluate: $\int \frac{3x+1}{\sqrt{5-2x-x^2}}\,dx$

SECTION – C

20. Let $A=\{1,2,3.....,9\}$ and R be the relation in $A\times A$ defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d $\in$ A.

    Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

OR

Let $f : N \to R$ be a function defined as $f(x)=4x^2+12x+15$.

Show that $f : N \to S$ is invertible, where S is the range of f. Hence inverse of f.

21. Compute, using integration, the area bounded by the lines.

    $x+2y=2 \quad y-x=1$ and $2x+y=7$
22. Find the particular solution of the differential equation $xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{y}{x}\sin\left(\frac{y}{x}\right)=0$, given that $y=0$, when $x=1$

OR

Obtain the differential equation of all circles of radius r.

23. Show that the lines $\vec{r}=\left(-3\hat{i}+3\hat{j}+5\hat{k}\right)+\lambda\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\mu\left(-\hat{i}+2\hat{j}+5\hat{k}\right)$ are coplanar. Also, find the equation of the plane containing these lines.
24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected students was a hostelier?
25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed 80km/h, the petrol cost increases to R. 3 per km. He has almost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel.
26. A jet of energy of flying along the curve $y=x^2+2$ and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet.

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