

FULL LENGTH TEST - 2

1. All questions are compulsory
2. Question numbers 1 to 6 carry 1 mark each.
3. Question numbers 7 to 19 carry 4 marks each
4. Question numbers 20 to 26 carry 6 marks each.

SECTION - A

1. Evaluate: $\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right)$.
2. State the reason for the following Binary Operation *, defined on the set Z of integers, to be not commutative. $a*b = ab^3$
3. Give an example of a skew symmetric matrix of order 3.
4. Using derivative, find the approximate percentage increase in the area of a circle if its radius is increased by 2%
5. Find the derivative of $f(e^{\tan x})$ w.r. to x at $x = 0$. It is given that $f'(1) = 5$
6. If the lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.

SECTION - B

7. Let $f : W \rightarrow W$ be defined as $f(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$ Then show that f is invertible.

Also, find the inverse of f .

OR

Show that the relation R in the set $N \times N$ defined by $(a, b)R(c, d)$ iff $a^2 + d^2 = b^2 + c^2 \forall a, b, c, d \in N$ is an equivalence relation.

8. Prove that, $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2}$, where

$$\pi < x < \frac{3\pi}{2}$$

9. Let $A = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$ Then verify the following

$A(\text{adj } A) = (\text{adj } A)A = |A|I$, where I is the identity matrix order 2.

10. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 3 & 4+3p & 2+4p+3q \\ 4 & 7+4p & 2+7p+4q \end{vmatrix} = 1$$

OR

Without expanding the determinant at any

stage, prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$

11. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute

AB. Hence, solve the following system of equations: $2x + y = 4$, $3x + 2y = 1$

12. If the following function is differentiable at $x = 2$, then find the values of a and b.

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 2 \\ ax + b, & \text{if } x > 2 \end{cases}$$

13. Let $y = (\log x)^x + x^{x \cos x}$ Then find $\frac{dy}{dx}$

OR

if $x = a \sin pt$, $y = b \cos pt$. Then find $\frac{d^2y}{dx^2}$ at $t = 0$.

14. Evaluate the following indefinite integral:

$$\int \frac{1}{\sin x - \sin 2x} dx.$$

OR

Evaluate the following indefinite integral:

$$\int \frac{\sin \phi}{\sqrt{\sin^2 \phi + 2\cos \phi + 3}} d\phi$$

15. Evaluate the following definite integral:

$$\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx.$$

16. Solve the following differential equation:

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}, x > 0$$

17. Solve the following differential equation:

$$(1 + y^2)dx = (\tan^{-1} y - x)dy.$$

18. Find the shortest distance between the following pair of skew lines:

$$\frac{x-1}{2} = \frac{2-y}{3} = \frac{z+1}{4}, \frac{x+2}{-1} = \frac{y-3}{2} = \frac{z}{3}.$$

19. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{2}{3}$. What is probability that

- (i) the problem will be solved?
- (ii) at most one of them will solve the problem?

SECTION – C

20. Find the intervals in which the following function is strictly increasing or strictly decreasing. Also, find the points of local maximum and local minimum, if any.

21. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$

and the angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$. then

prove that

(i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$,

(ii) $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = \pm 1$

22. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the points (2, 1) and the lines whose equation are $x = 2y$ and $x = 3y - 3$

23. Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel

to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the

foot of the perpendicular from the given point upon the given plane.

OR

Find the equation of the plane passing through the line of intersection of the planes

$$r \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = -1 \text{ and } r \cdot (\hat{i} + \hat{j} - \hat{k}) = 0 \text{ and}$$

passing through the point (3, -2, -1) Also,

find the angle between the two given planes.

24. A bag I contains 5 red and 4 white balls and a Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II. If the Ball drawn from the Bag II is red, then find the probability that one red ball and one white ball are transferred from the Bag I to Bag II.

OR

Find the mean, the variance and the standard deviation of the number of doublets in three throws of a pair of dice.

25. A farmer wants to construct a circular garden and a square garden in his field. He wants to keep the sum of their perimeter 600 m. Prove that the sum their areas is the least, when the side of the square garden is double the radius of the circular garden.

26. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 hours of labour for fabricating and 1 hour for finishing. Each piece of model B requires 12 hours of labour for fabricating and 3 hours for finishing. The maximum number of labour hours, available for fabricating and for finishing, are 180 and 30 and respectively. The company makes a profit of Rs 8000 and Rs. 12000 on each piece of model A model B respectively. How many pieces of each model should be manufactured to get maximum profit? Also, find the maximum profit.