FULL SYLLABUS TEST PAPER

Time : 3 hours. **MM: 100**

General Instruction

1. All questions are compulsory.
2. Please check that this Question paper contains 26 Questions.
3. Marks for each question are indicated against it.
4. Questions 1 to 6 in Section – A are Very Short Answer Type Questions carrying **one mark** each.
5. Questions 7 to 19 in Section – B are Long Answer I Type Question carrying **4 marks** each.
6. Questions 20 to 26 in Section – C are Long Answer II Type Questions carrying **6 marks** each.
7. Please write down the serial number of the Question before attempting it.

---

1. If A is a square matrix satisfying $A^2 = I$, then what is the inverse of A?
2. Show that the points $A(3, -5, 1)$, $B(-1, 0, 8)$ and $C(7, -10, -6)$ are collinear.
3. Find the equation of the plane passing through $(2, 3, -1)$ and perpendicular to the vector $3\hat{i} - 4\hat{j} + 7\hat{k}$.
4. Determine the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right) = \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}$
5. Evaluate: $\int_{-\pi/2}^{\pi/2} \sin x \cos^2 x \left(\sin^2 x + \cos x\right) dx$
6. Evaluate: $\int \frac{dx}{\sqrt{9 - 25x^2}}$
7. Find $\frac{dy}{dx}$, when $y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$, where a is a constant.
8. Differentiate the function with respect to x:

   $\log\left(\frac{a + b\sin x}{a - b\sin x}\right)$
9. Find the probability of drawing a diamond card in each of the two consecutive draws from a well shuffled pack of cards, if the card drawn is not replaced after the first draw.

   OR

   A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution and means of numbers of tails. Is this a good tendency? Justify your answer.
10. Prove that: $2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right) = \tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\sin\beta + \cos\alpha}\right)$
11. Find the interval in which the value of the determinant of the matrix A lies.

    Given $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

12. Prove that: $\int_0^{\pi} x\sin^3 x\,dx=\frac{2\pi}{3}.$

OR

Evaluate: $\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)\,dx$

13. Find the length and the coordinates of the foot of the perpendicular from the point $(7,14,5)$ to the plane $2x+4y-z=2$.

14. Let $A=\{1,\ 2,\ 3,\ .....,\ 9\}$ and R be the relation in $A\times A$ defined by $(a,b)\ R\ (c,d)$ if $a+d=b+c$ for $(a,\ b),(c,d)$ in $A\times A$. Prove that R is an equivalence relation.

OR

let '*' be a binary operation in the set $\{0,1,2,3,4,5\}$ and $a*b=\begin{cases} a+b, & \text{if } a+b<6\\ a+b-6 & \text{if } a+b\geq 6\end{cases}$

Find the identity element and the inverse element of each element of the set for operation '*'

15. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 5 & 3\\ 0 & 2 & 1\end{bmatrix}$, then find A^{-1}, using elementary row operations.

16. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c},\ \vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$, then prove that $\vec{b}=\vec{c}$.

17. Evaluate $\int\frac{3x+1}{\sqrt{5-2x-x^2}}\,dx.$

18. Determine the values of $a,\ b$ and c for which the function

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, & x>0\end{cases}$$

may be continuous at $x=0$.

OR

It is given for the function $f(x)=x^3+bx^2+ax+5$ on $[1,3]$, Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$.

Find the values of a and b.

19. Solve the differential equation: $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

20. Show that the cone of greatest volume which can be inscribed in a given sphere is such that three times its altitude is twice the diameter of the sphere. Find the volume of the largest cone inscribed in a sphere of radius R.

21. Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards. A random variable X denotes the number of hearts in the three cards drawn. Find the mean and variance of X.

OR

Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

U79/17, DLF City Phase – III, Gurgaon – 122002 | Tel : +91 124 3926025, 96509 12288
web : www.equalsign.in | www.onlinepadho.com | email : info@equalsign.in

22. A factory owner wants to purchase two types of machines A and B, for his factory. The machine A requires an area of $1000\ \text{m}^2$ and 12 skilled men for running it and its daily output is 50 units, whereas the machine B requires $1200\ \text{m}^2$ area and 8 skilled men, and its daily output is 40 units. An area of $7600\,\text{m}^2$ and 72 skilled men be available to operate the machines.
    (i) How many machines of each type should be bought to maximize the daily output?
    (ii) Write two advantages of keeping skilled men in a firm.
23. Slow that the lines $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ and $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ are coplanar. Also, find the equation of the plane containing these lines.
24. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.
25. Find $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$

OR

Prove that the curves $y=x^2$ and $x=y^2$ divide the square bounded by $x=0$, $y=0$, $x=1$ and $y=1$ into three parts which are equal in area.

26. Find the particular solution of the differential equation $xe^{\frac{y}{x}}-y\sin\left(\frac{y}{x}\right)+x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)=0$, given that $y=0$, when $x=1$.